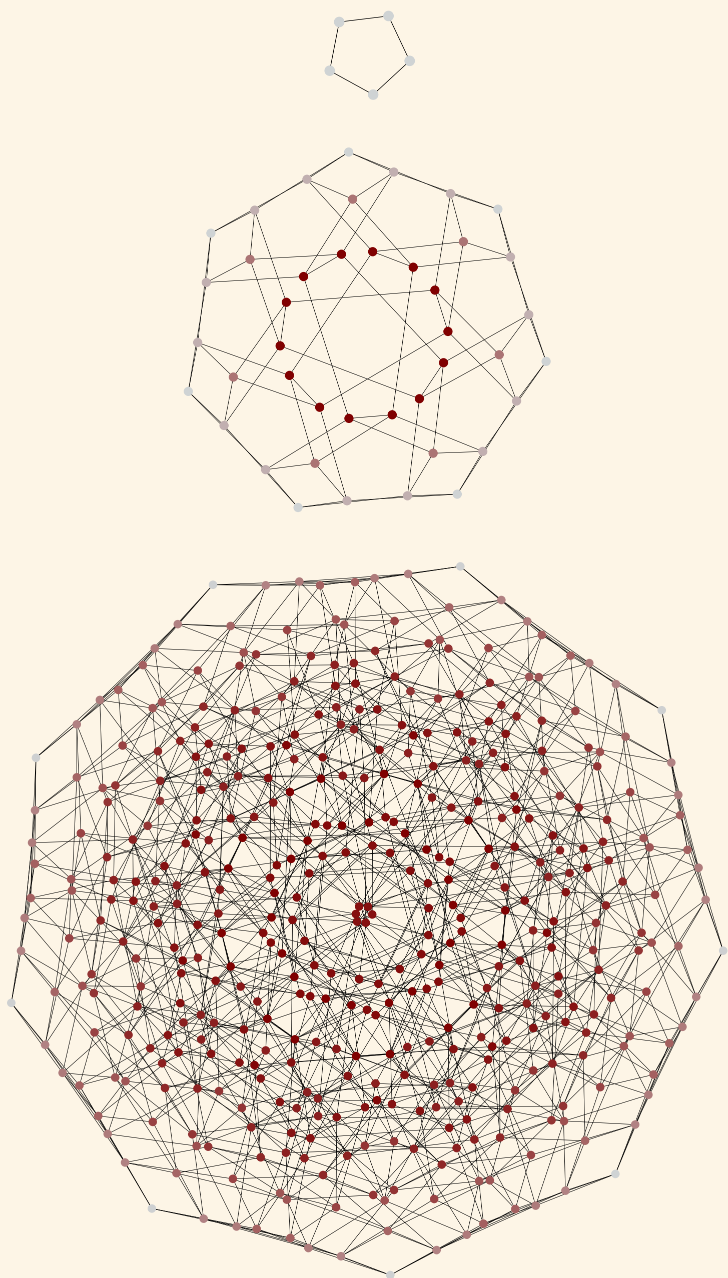
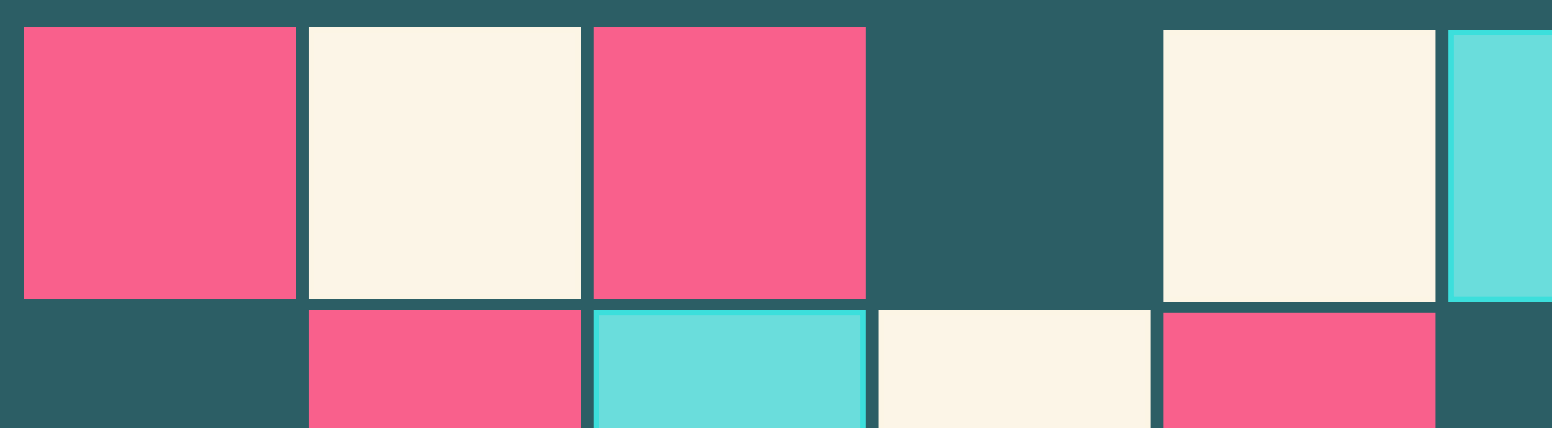
Surfaces, puzzles and moduli spaces

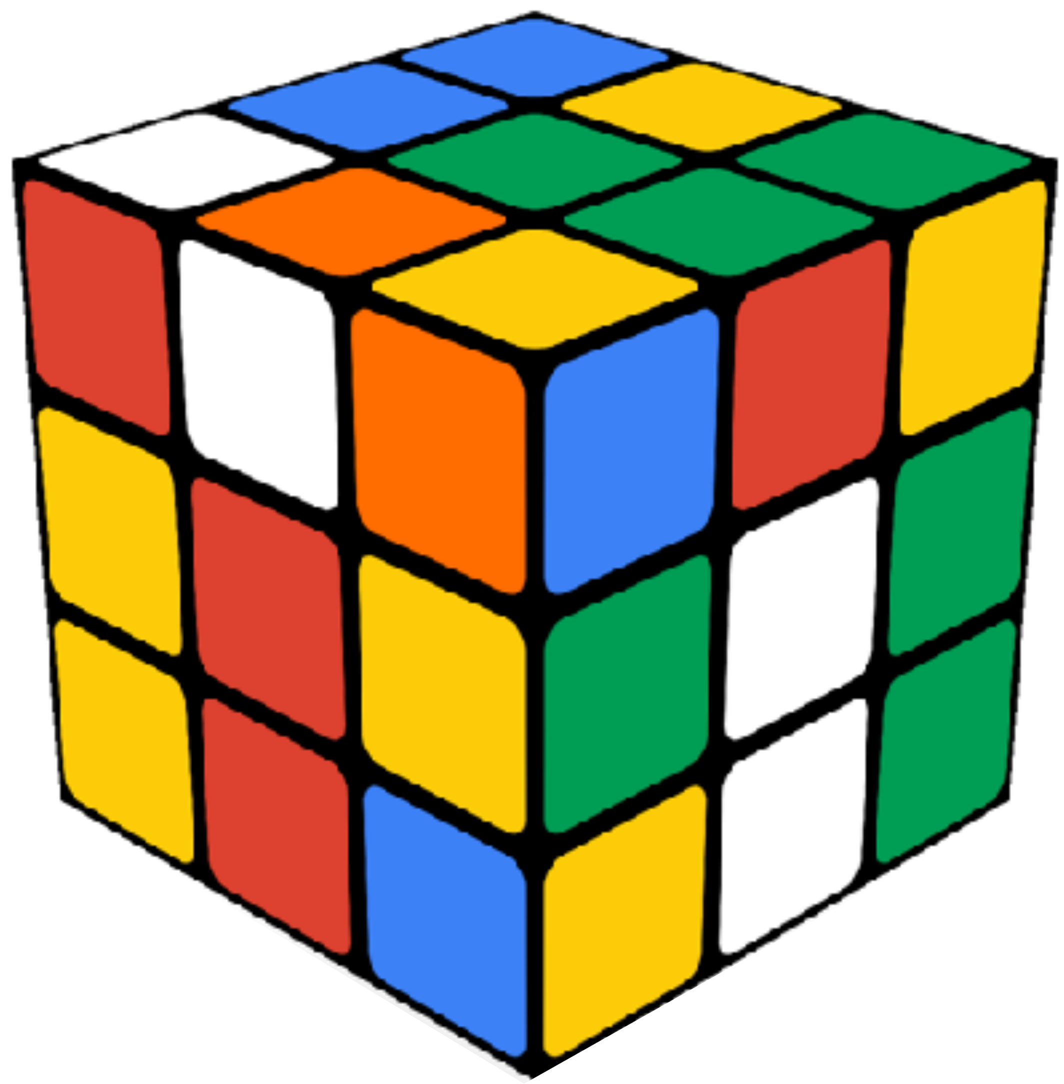


Hugo Parlier, *University of Luxembourg*

Collaborators

Paul Turner
Mario Guttierez
Mark Bell
Lionel Pournin

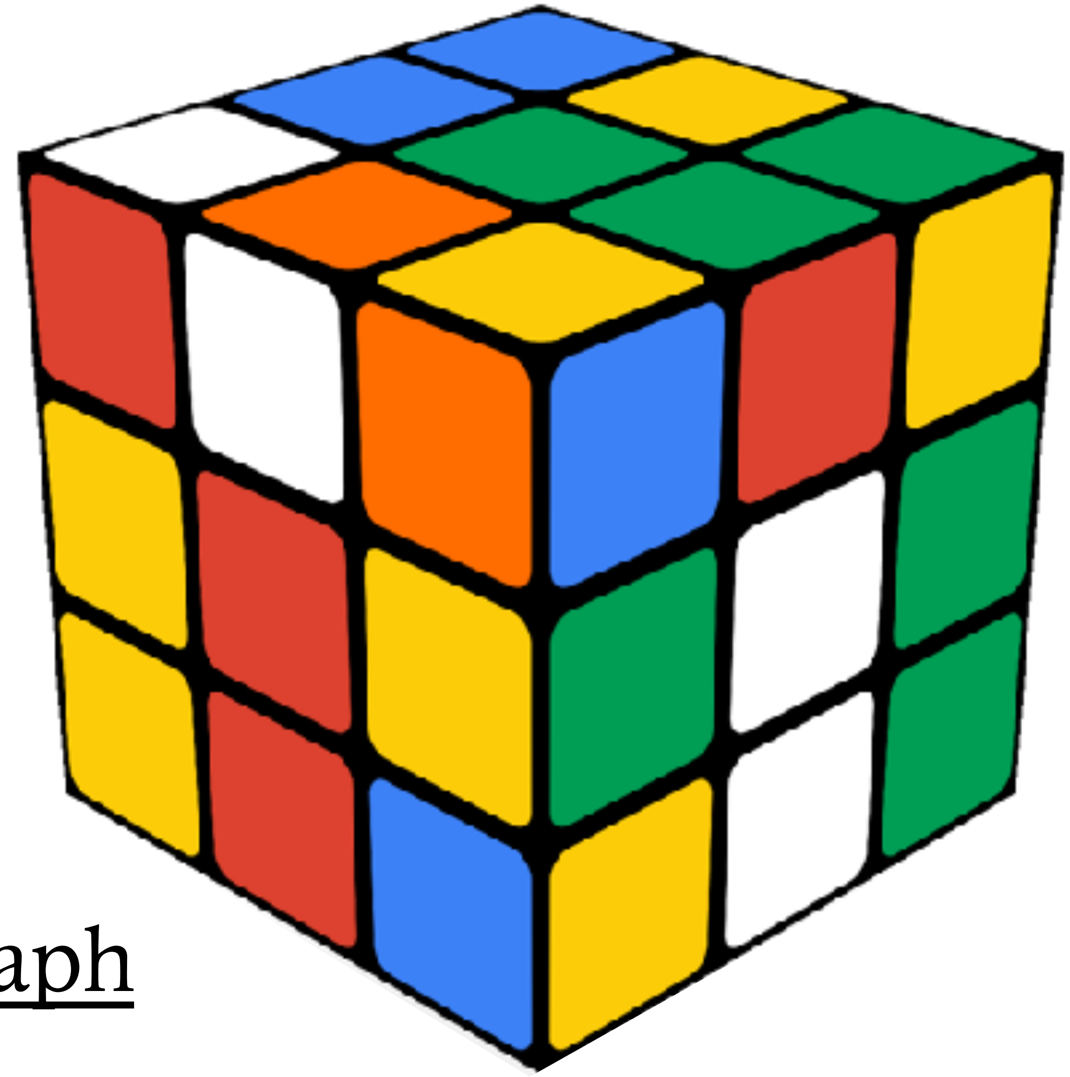




Distances between cube configurations

The distance between  and 

is the minimal number of moves between them

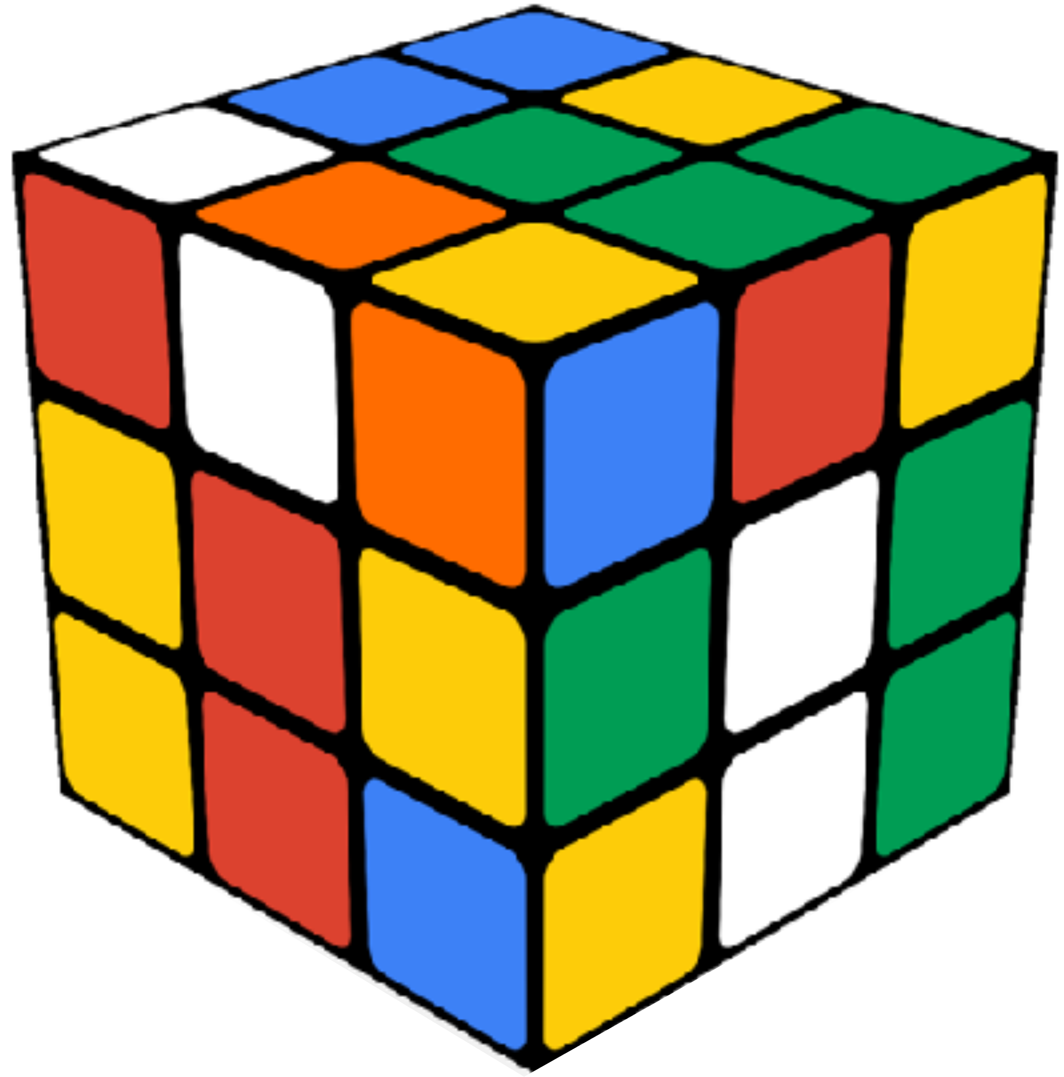


Rubik's cube graph

Vertices = configurations

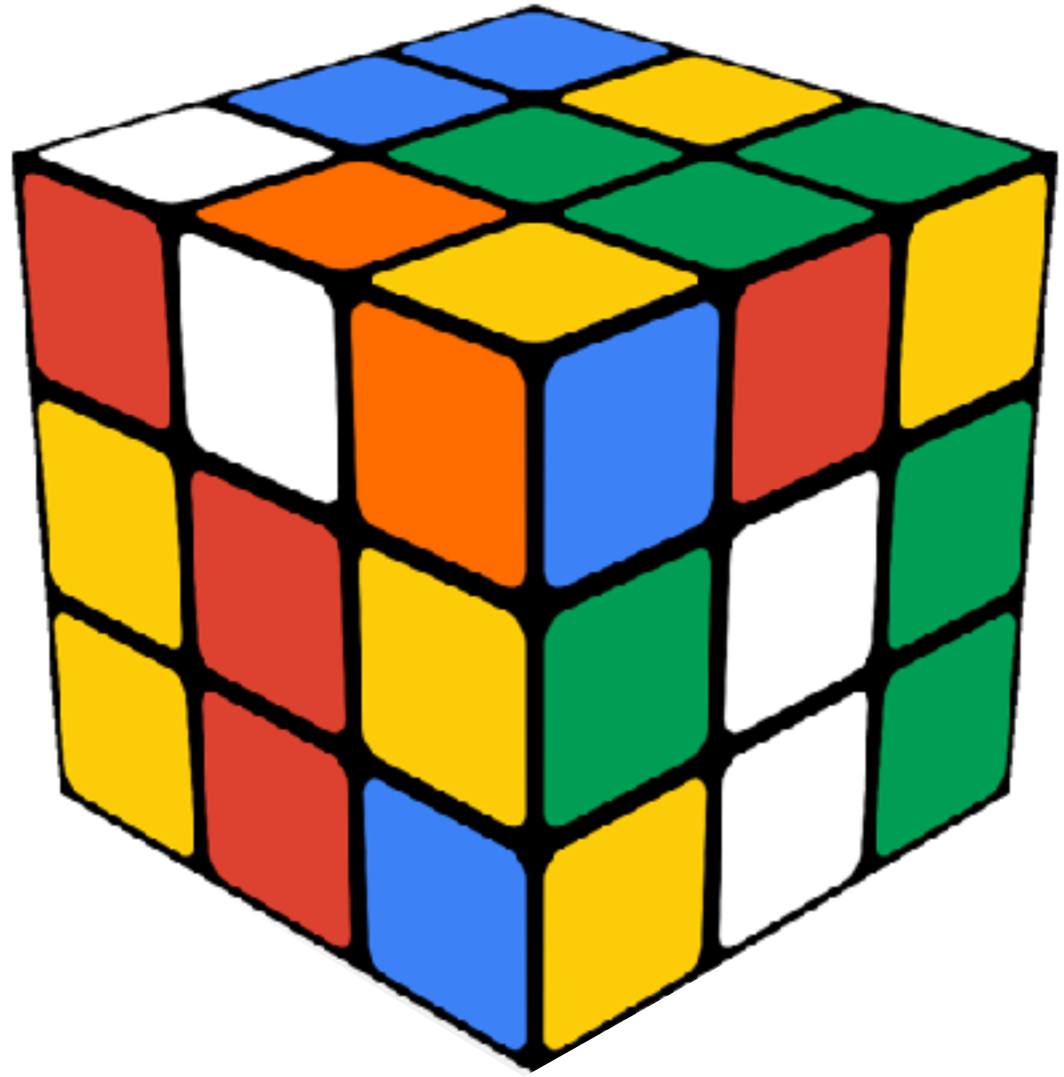
Edges = elementary moves (face rotations)

Understand the geometry (size, shape, …) of this configuration space.



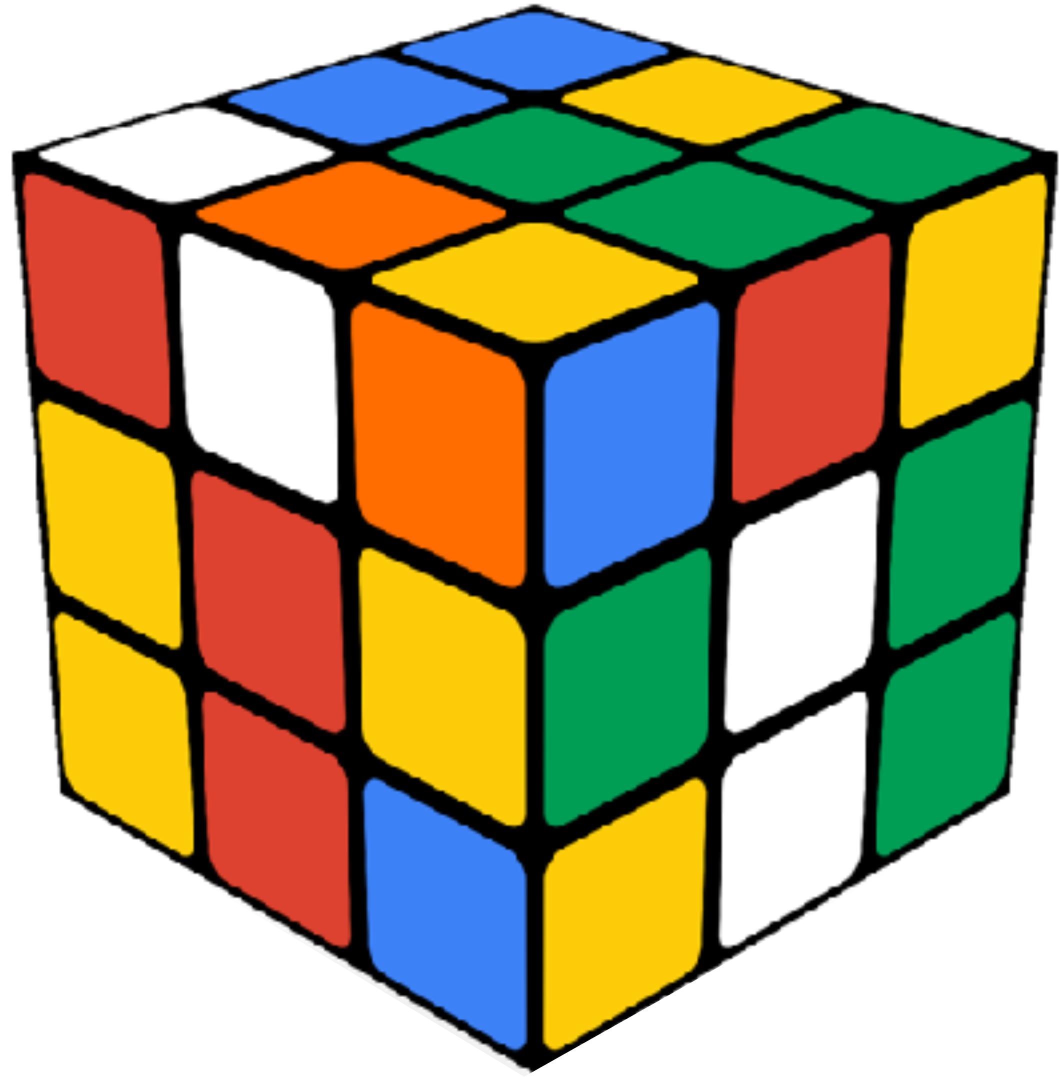
Theorem (Rokicki-Kociemba-Davidson-Dethridge, 2011)

The diameter of the (3 by 3 by 3) Rubik's cube graph is 20.



<u>Theorem</u> (Demaine-Demaine-Eisenstat-Lubiw-Winslow)

The diameter of the n by n by n Rubik's cube graph grows like $n^2/\log(n)$.

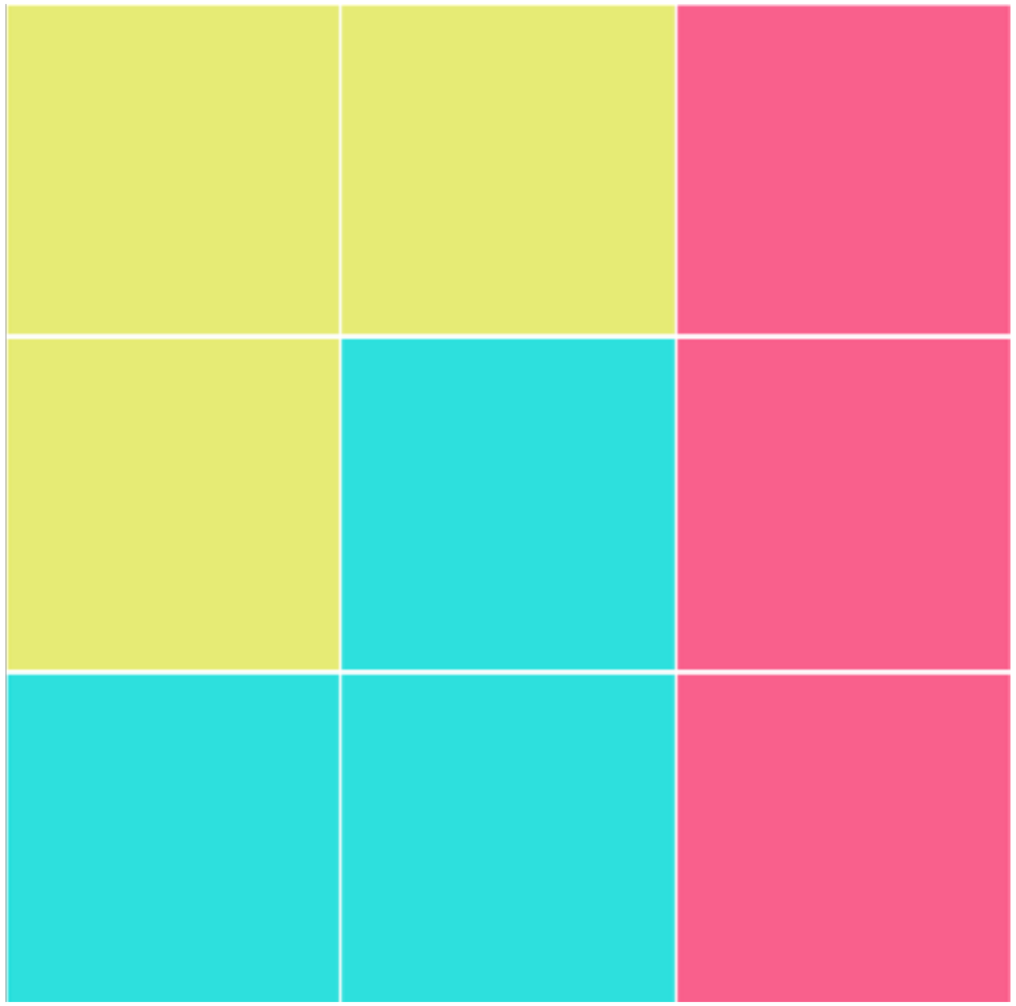


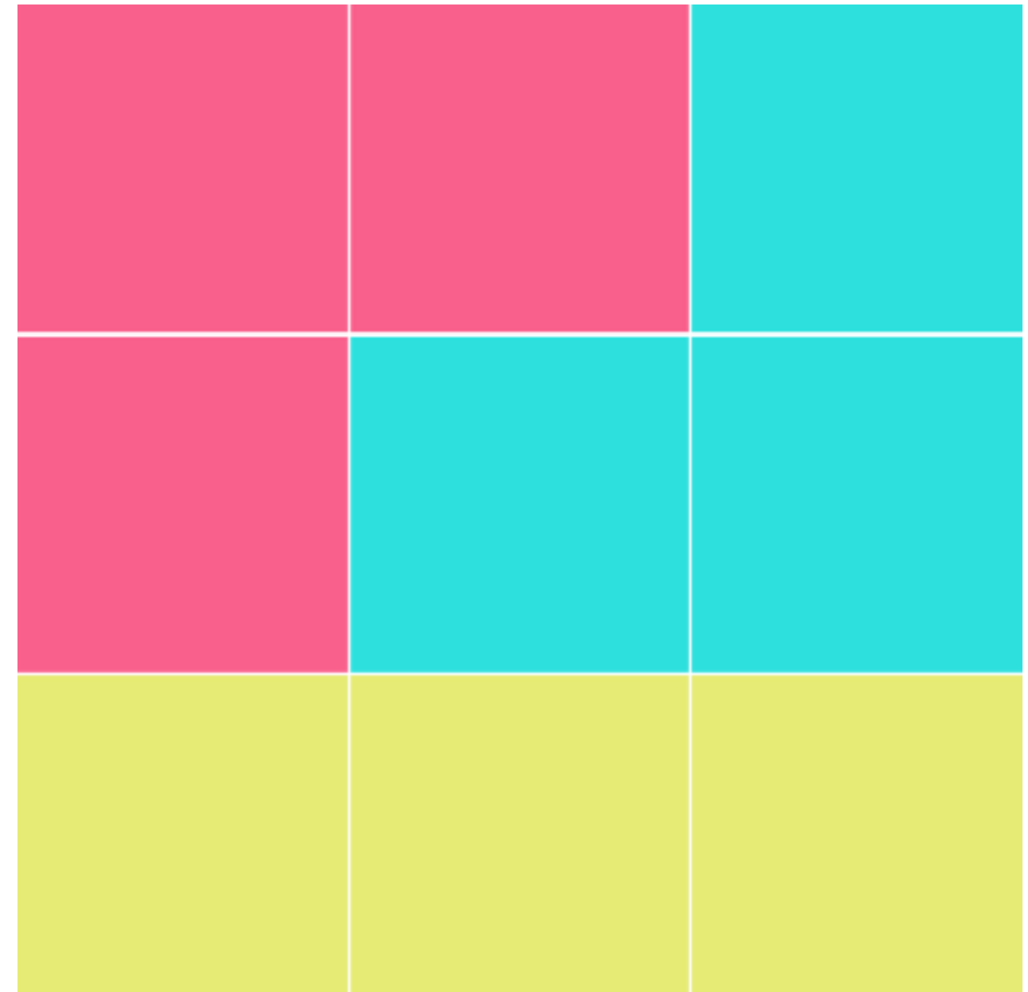
But what do Rubik's cube graphs look like?

The 3 by 3 by 3 has 43,252,003,274,489,856,000 vertices.

The 2 by 2 by 2 has 3,674,160 vertices.

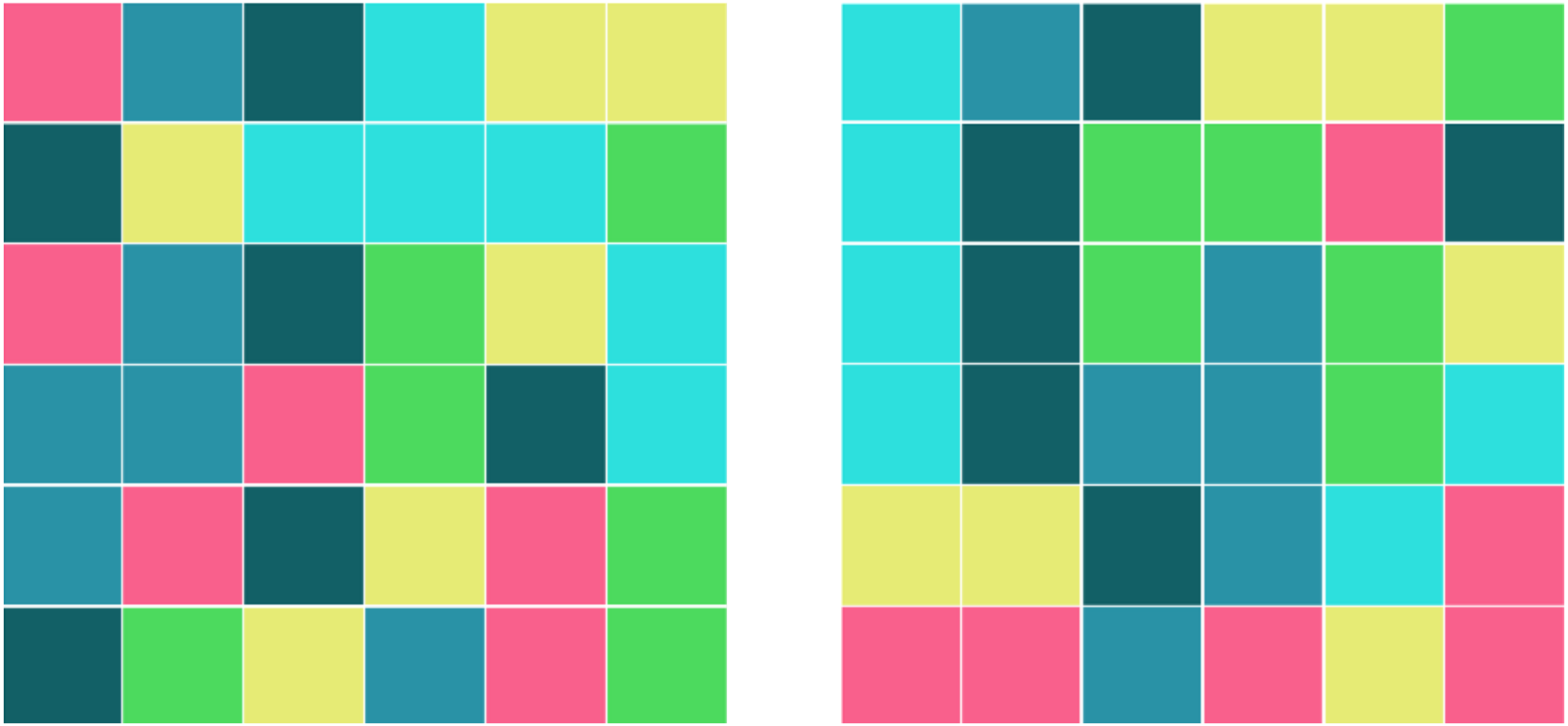
The only Rubik’s cube graph successfully visualized.





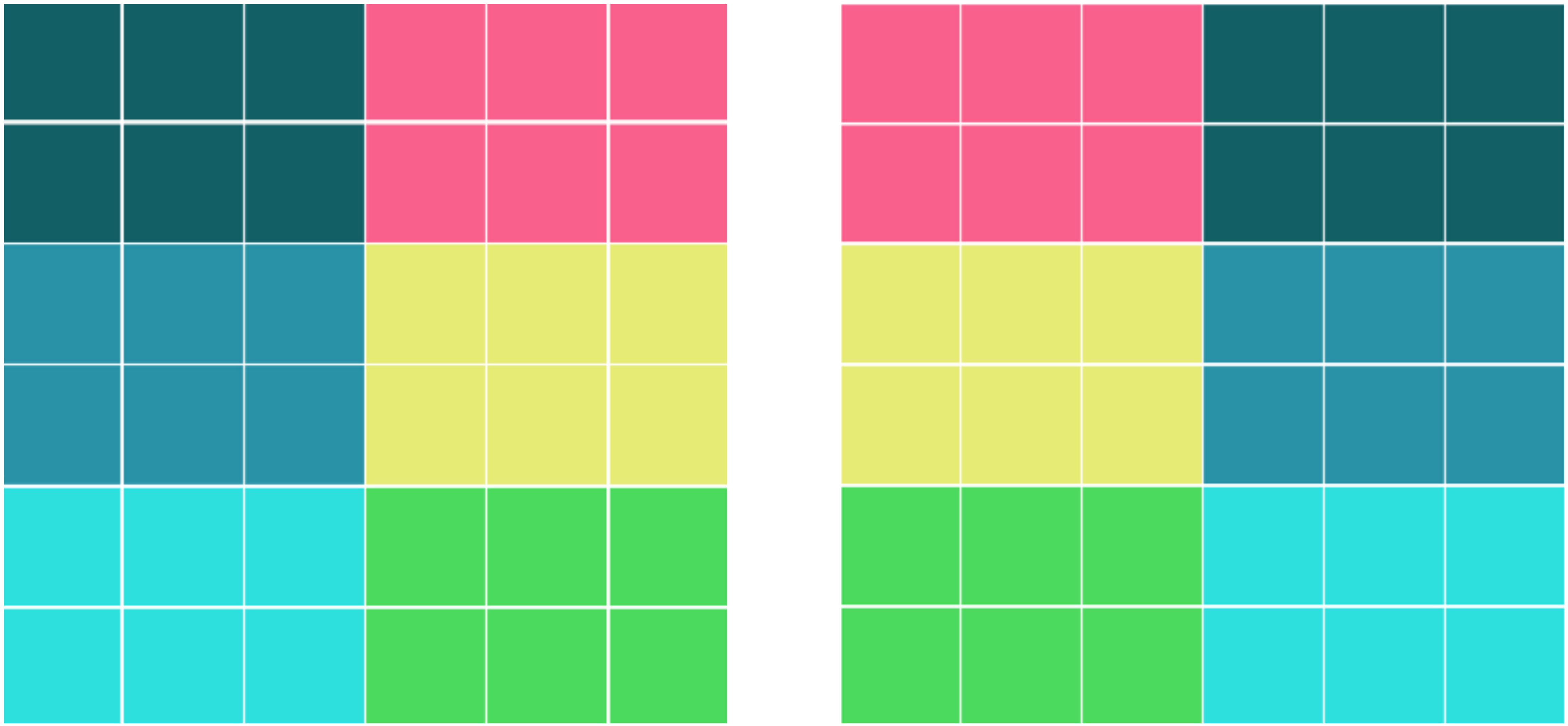
<u>Fact</u>

The diameter of the *3* by *3* Chroma-square graph is 6.



<u>Theorem</u> (P.-Turner)

The diameter of the n by n Chroma-square graph grows like n^2.



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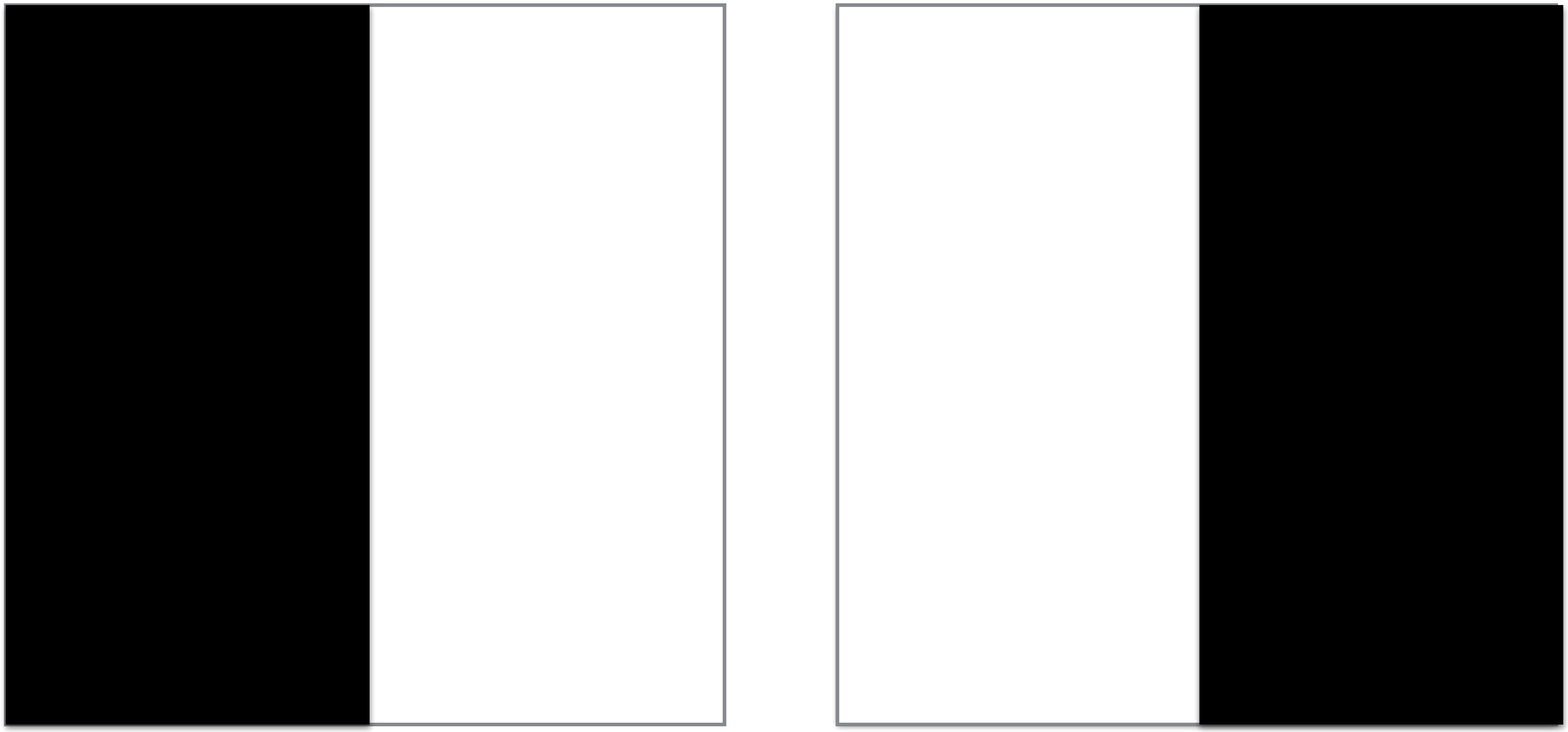
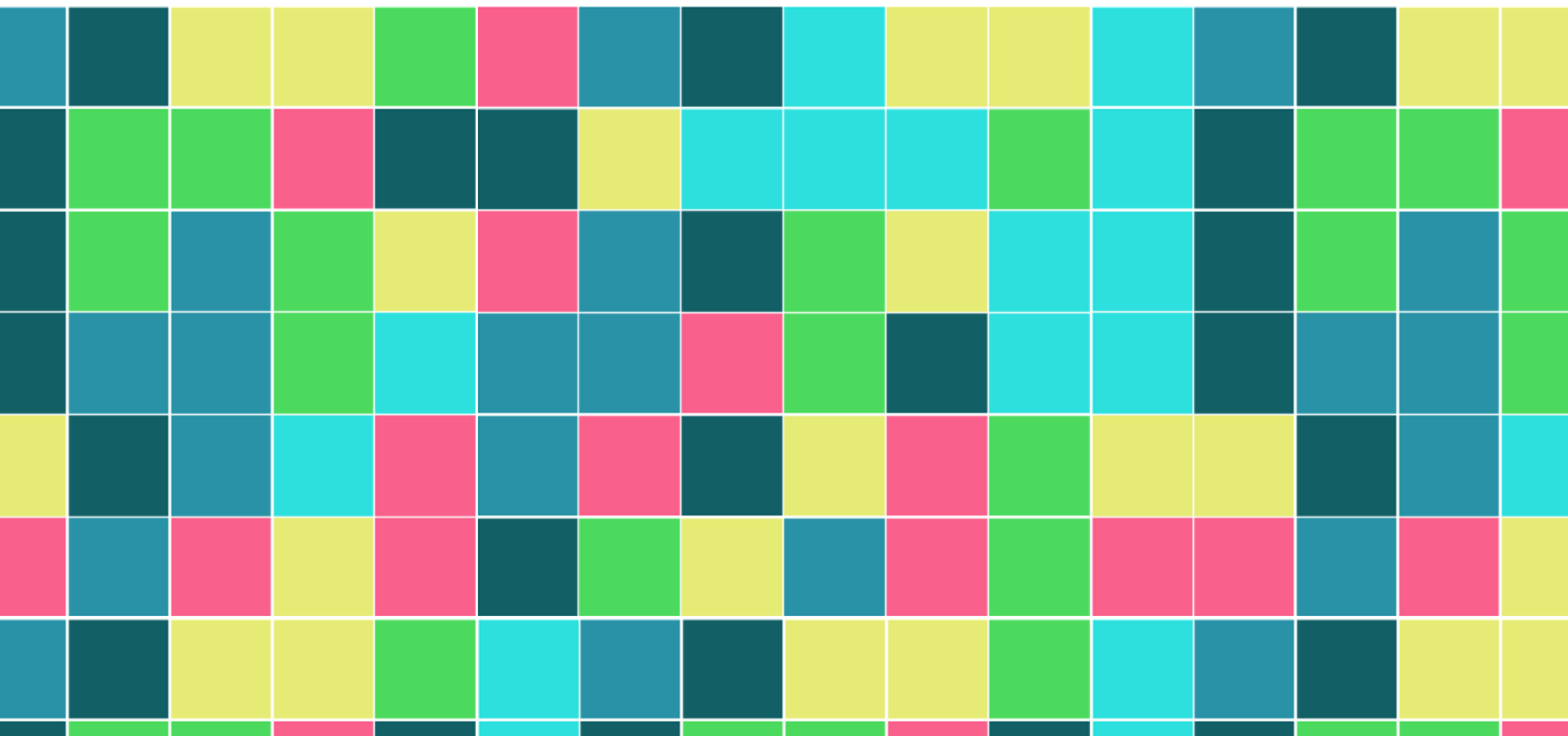


<u>Theorem</u> (P.-Turner)

The diameter of the n by n Chroma-square graph grows like n^2.

Fun facts about *arbitrary* colorings

- If there are at least 2 square tiles of the same color, the graph is connected.
- Diameter grows at most like n^2 for *any* coloring.



652 D. D. SLEATOR, R. E. TARJAN, AND W. P. THURSTON

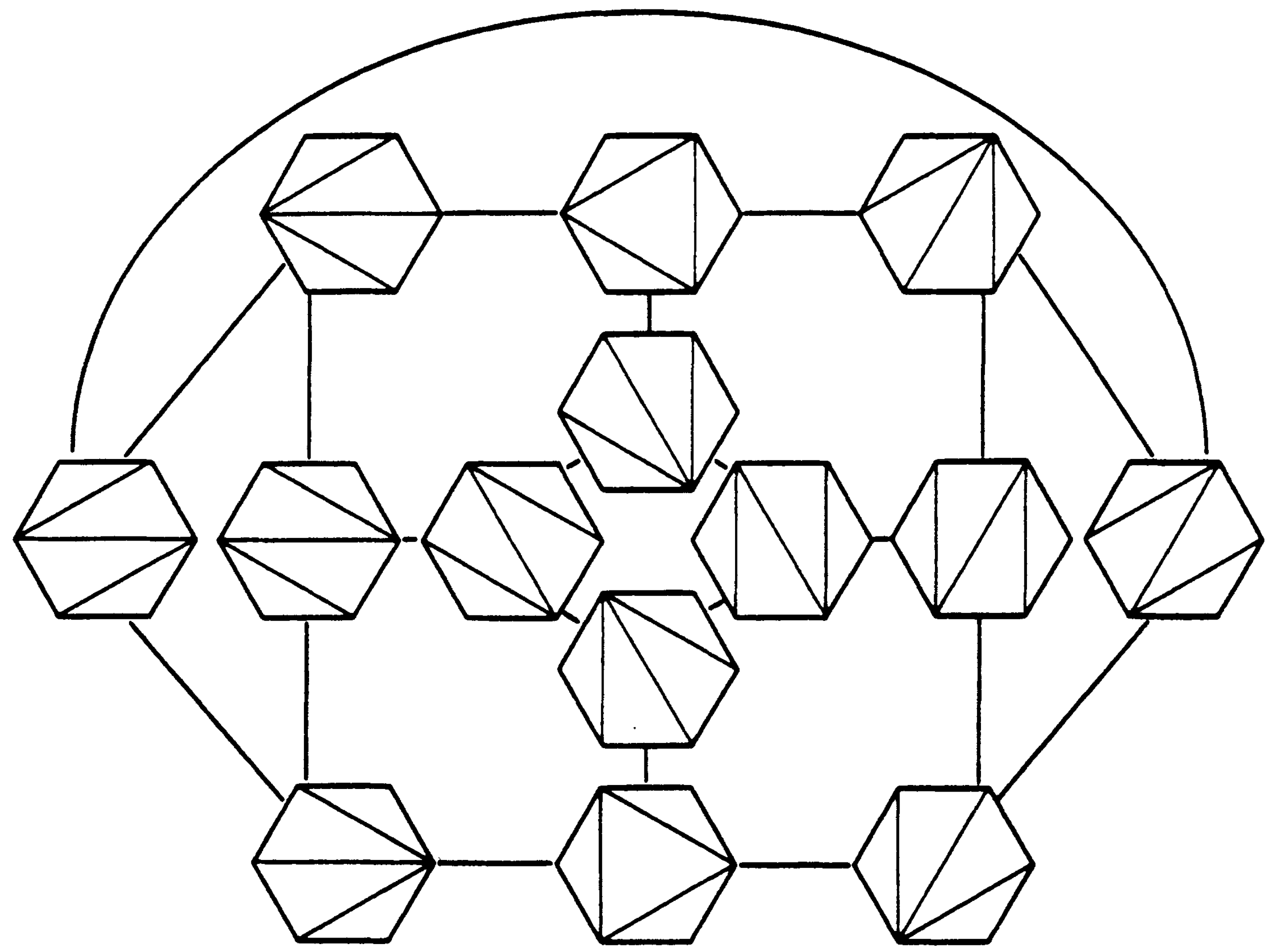
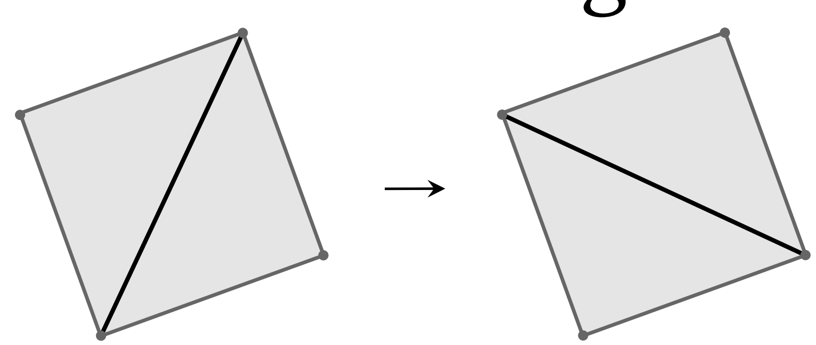


FIGURE 4. The rotation graph of a hexagon, $RG(6)$.

<u>Flip graphs of polygons</u> (Sleator-Tarjan-Thurston, 1988)

Vertices are triangulations and edges come from flipping diagonals:



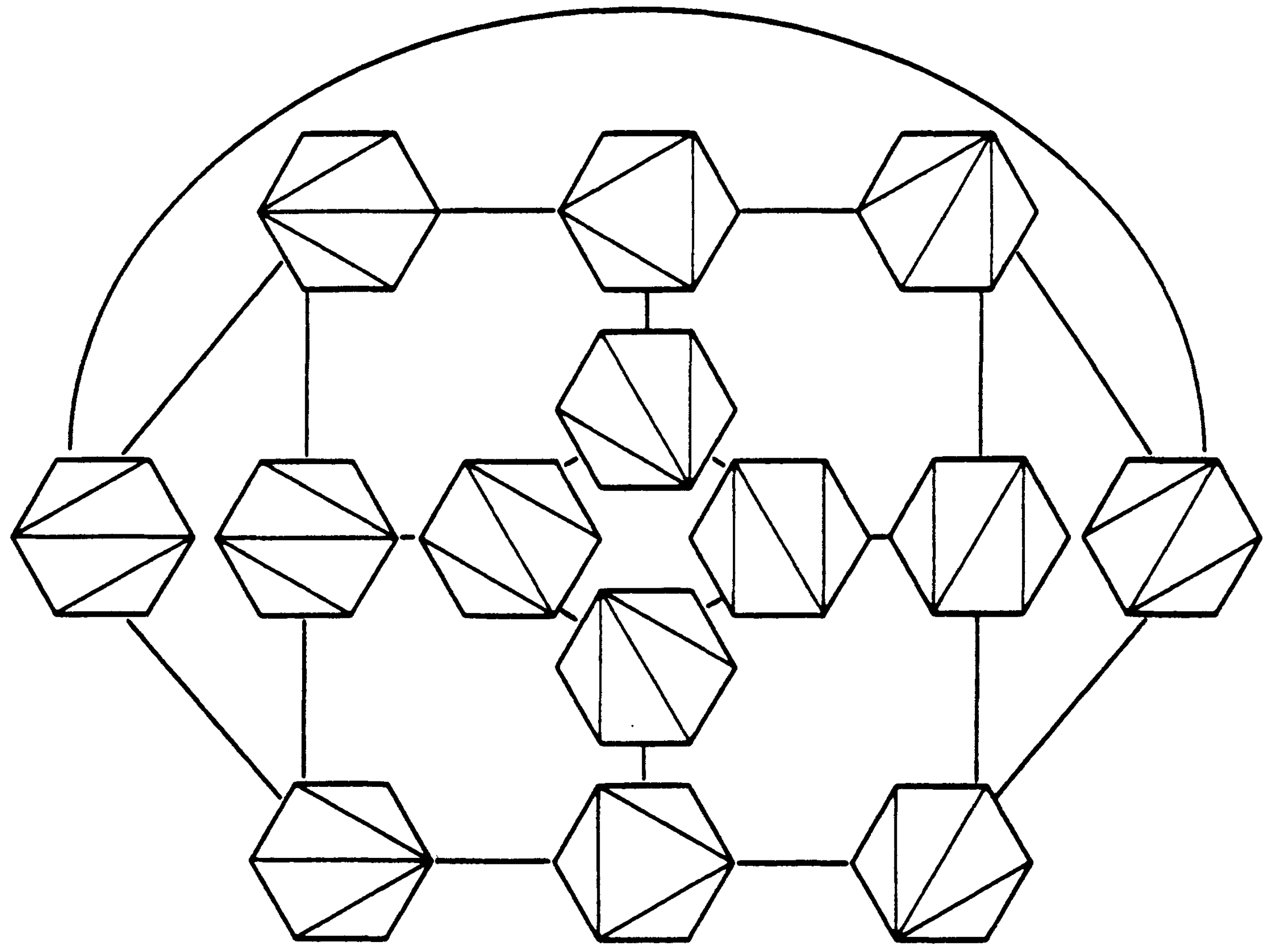


FIGURE 4. The rotation graph of a hexagon, $RG(6)$.

Theorem (Sleator-Tarjan-Thurston)

For sufficiently large n, the diameter of the flip graph of an n-gon is $2n$-10.

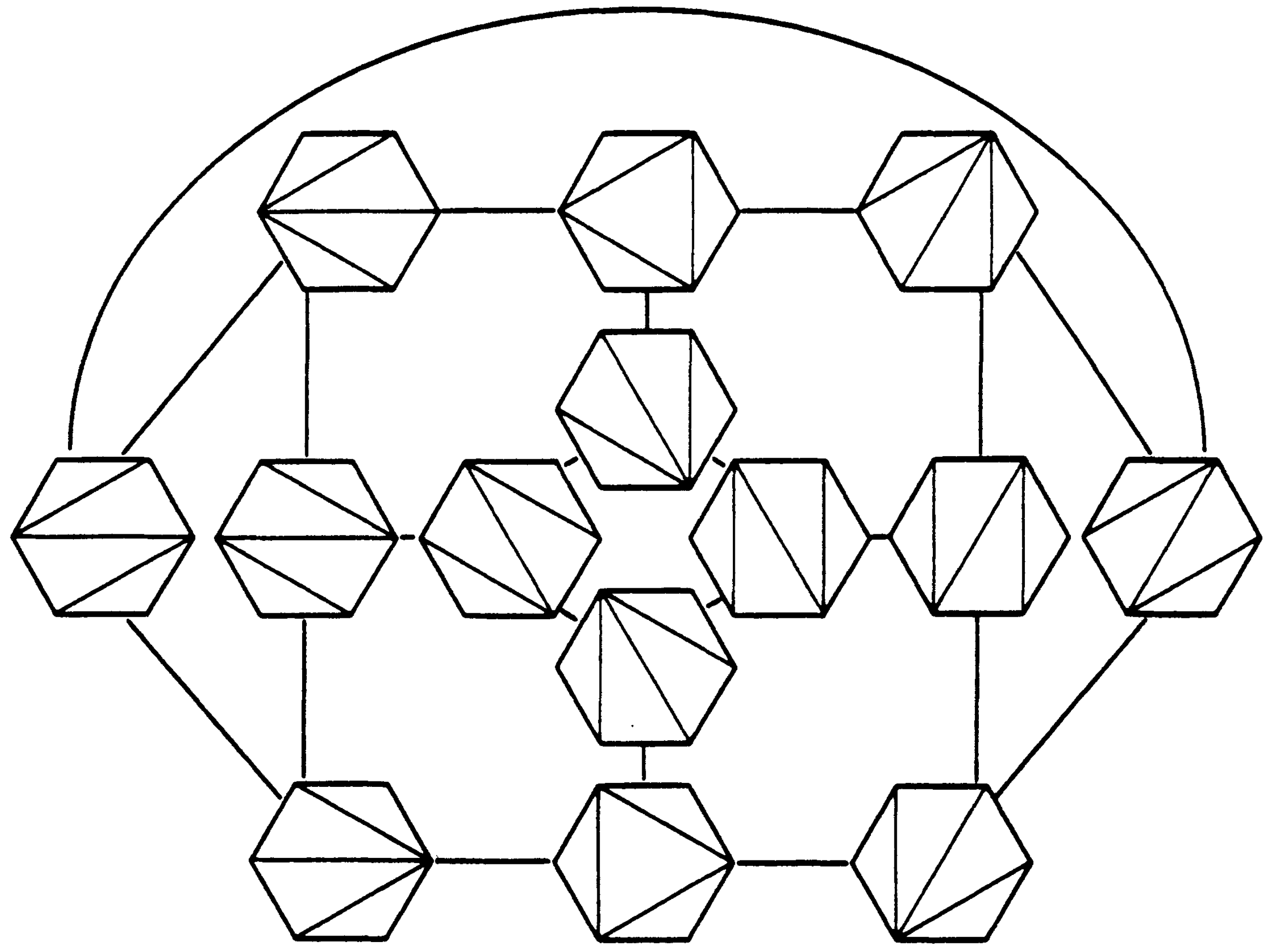


FIGURE 4. The rotation graph of a hexagon, $RG(6)$.

Theorem (Pournin)

For $n>12$ the diameter of the flip graph of an n-gon is $2n$-10.

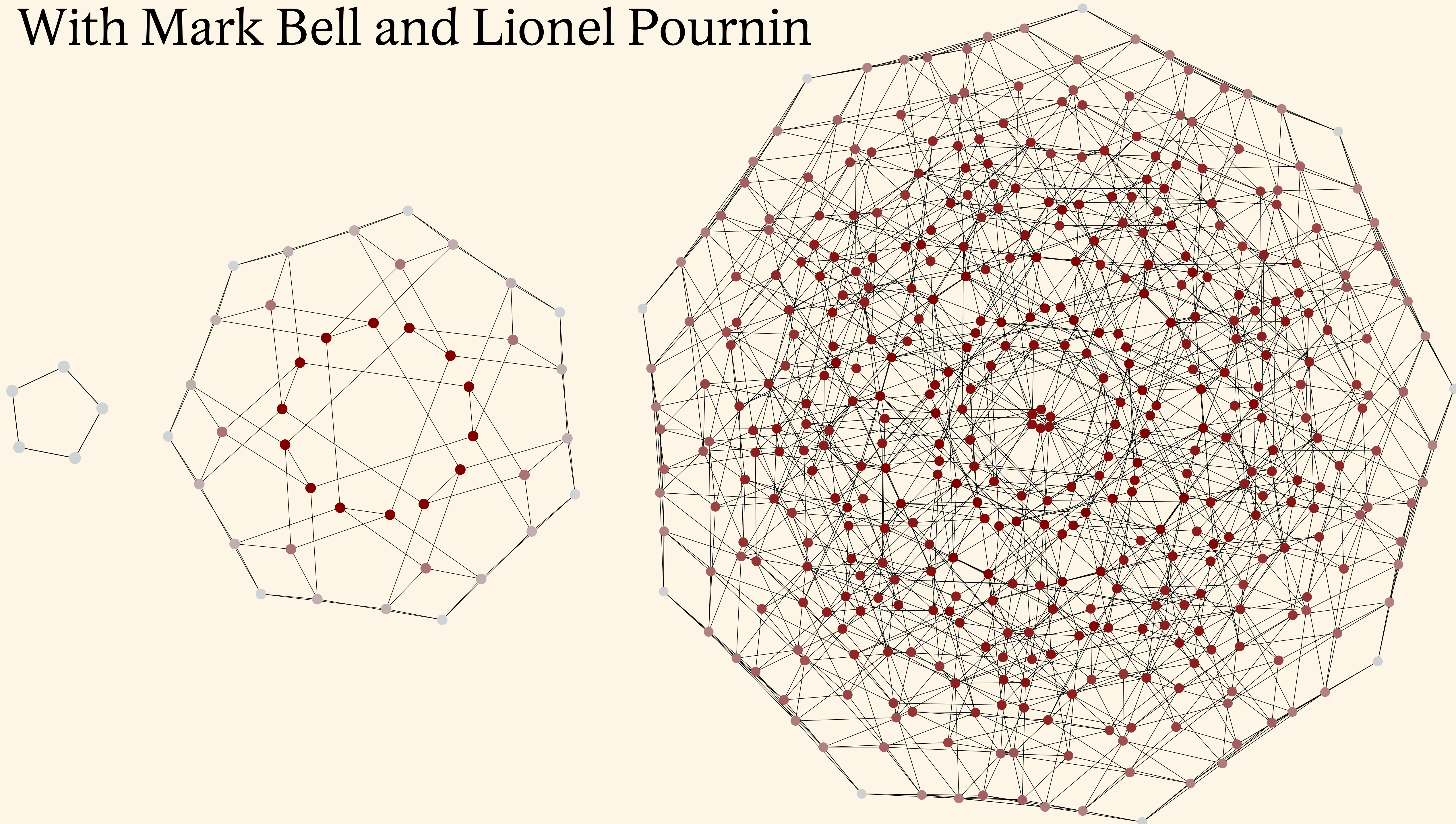
Visualizing Flip Graphs

With Mark Bell and Lionel Pournin

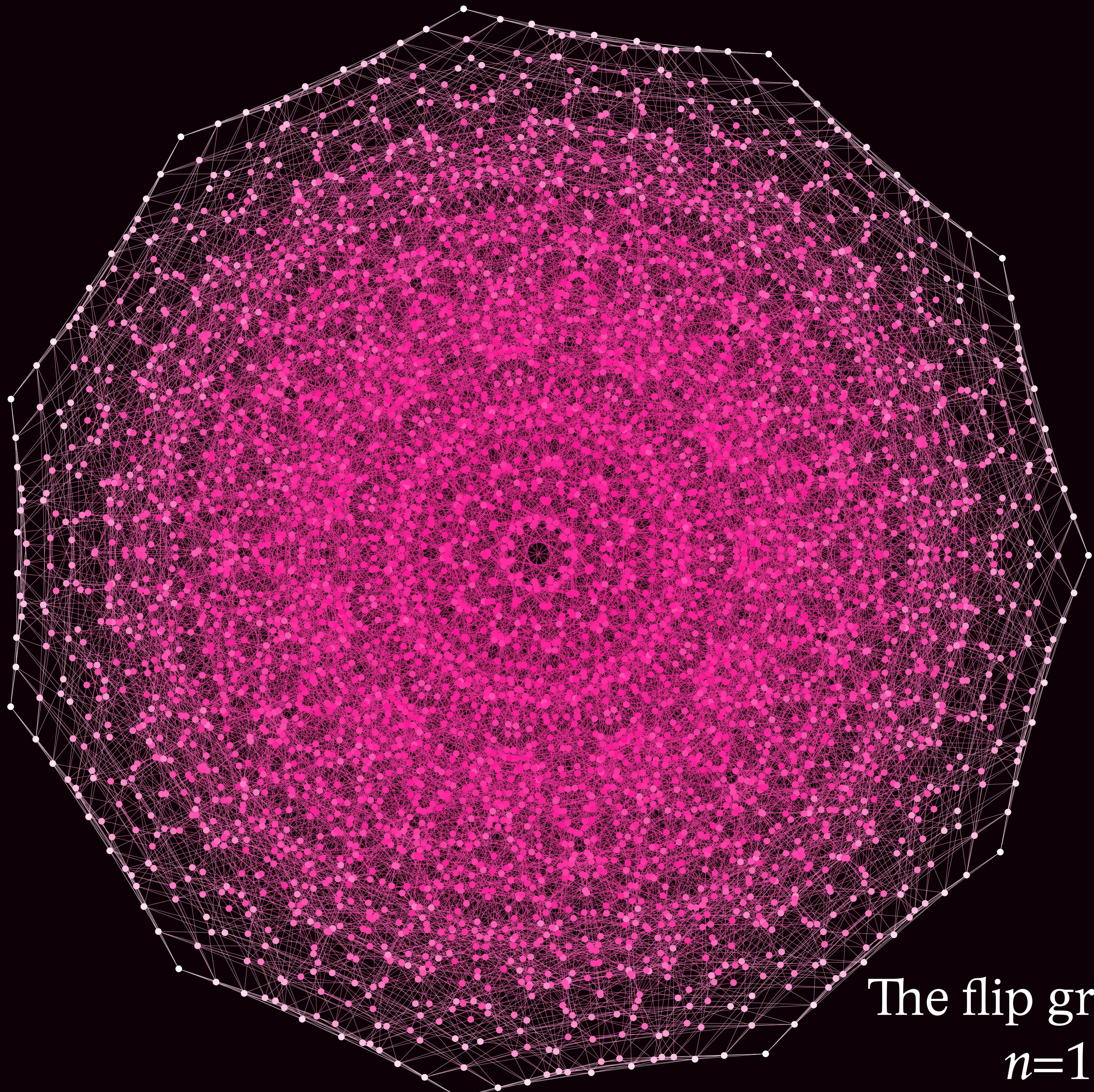
All visualizations made using Gephi and a visualization algorithm of Yifan Hu.

Visualizing Flip Graphs

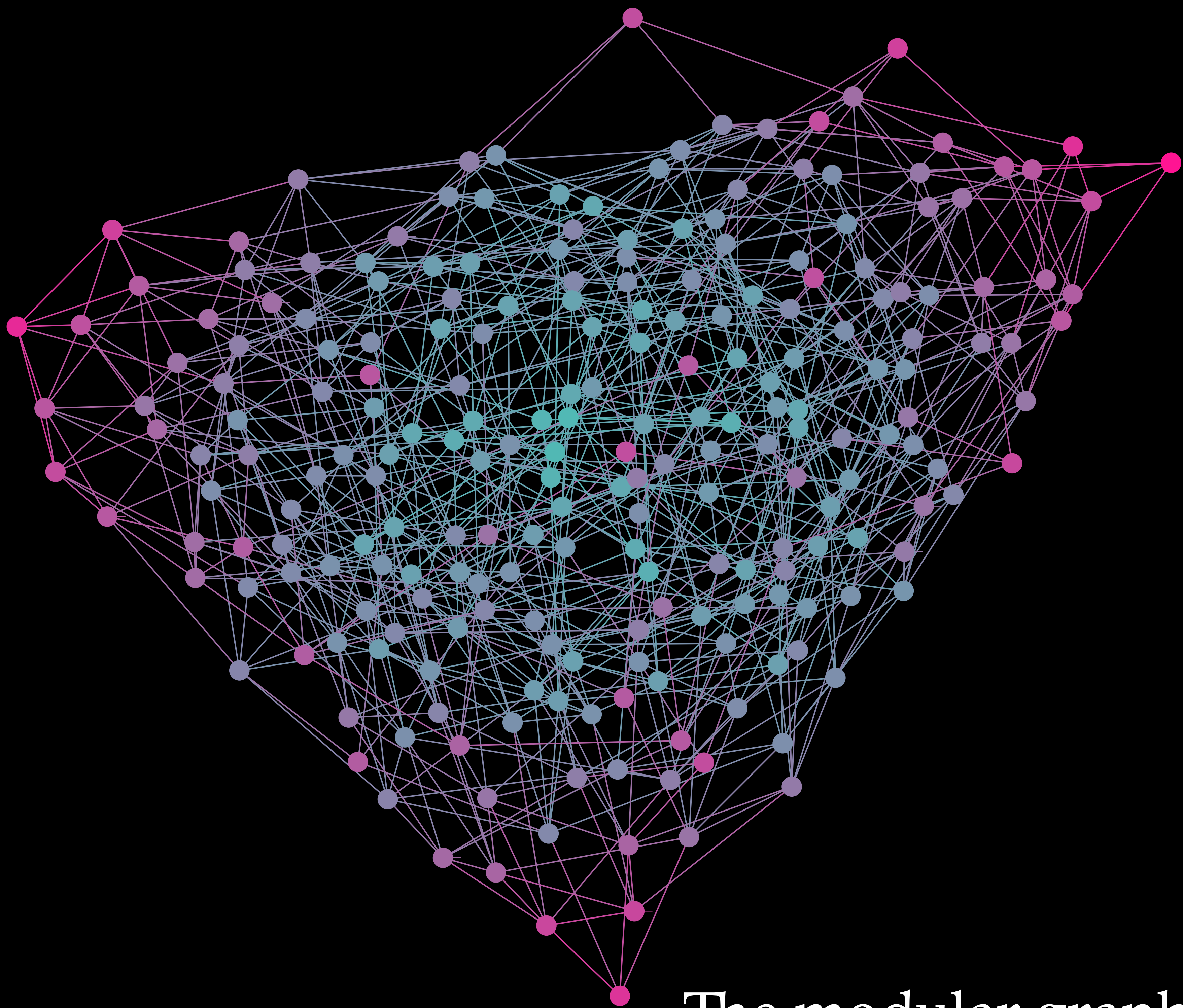
With Mark Bell and Lionel Pournin



The flip graphs of n-gons for n=5, 7 and 9

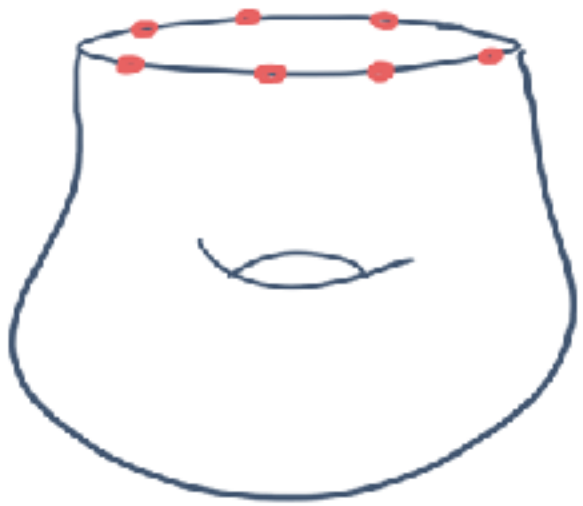


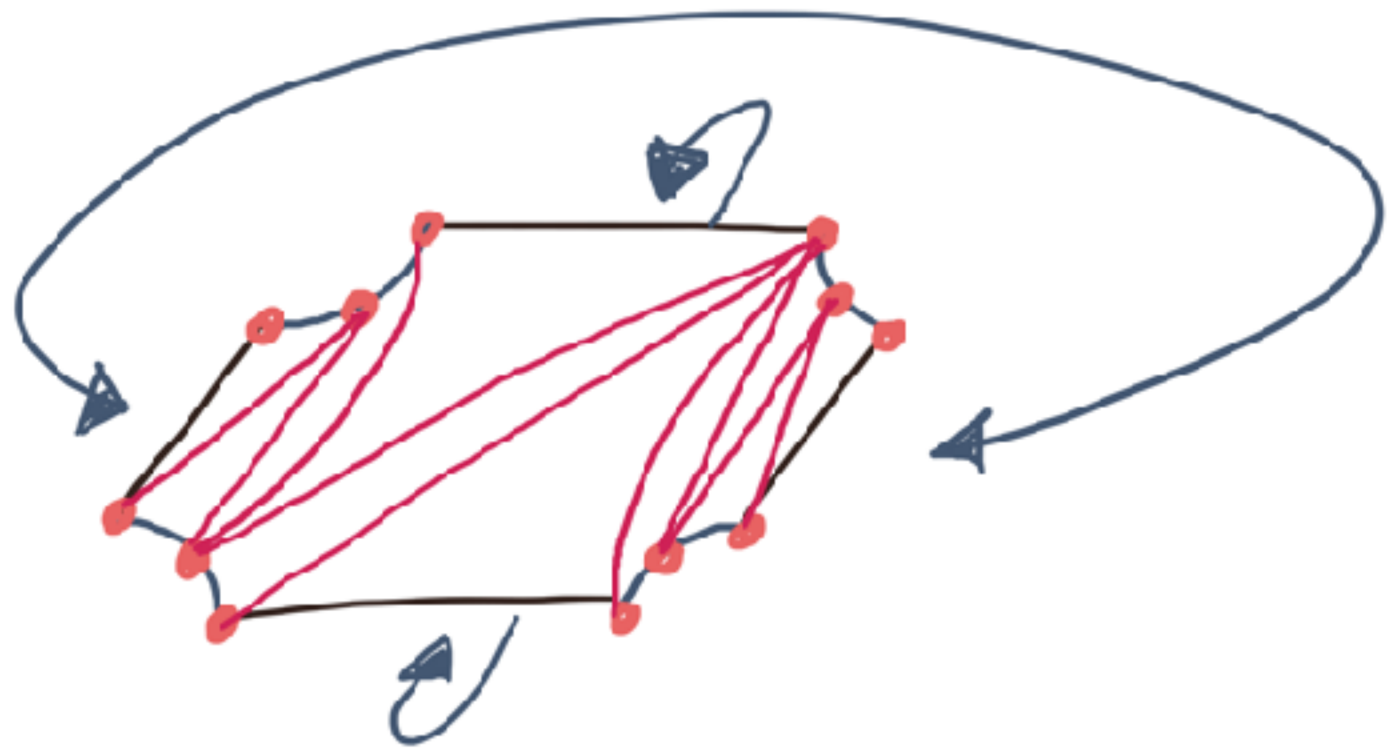
The flip graph for
n=11



The modular graph for n=11

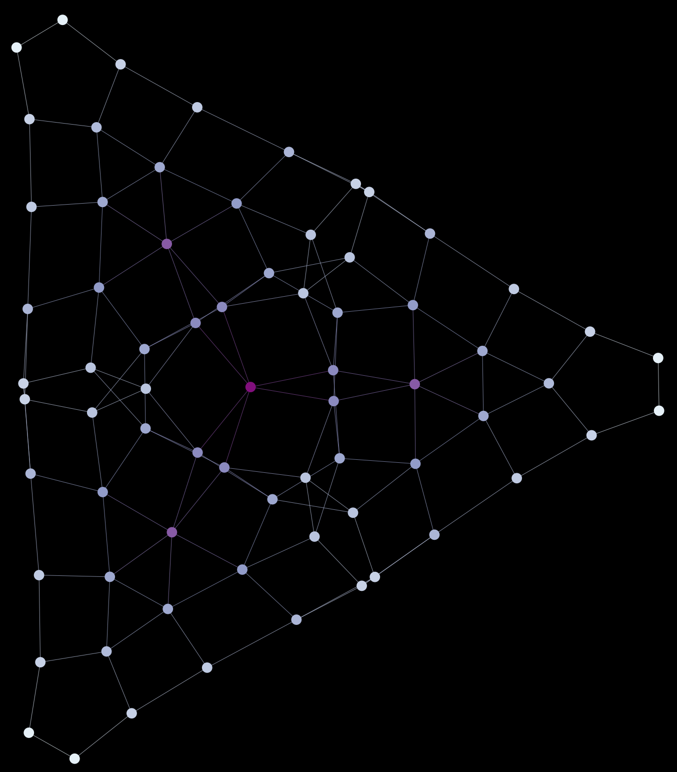
Flip Graphs of one-holed tori



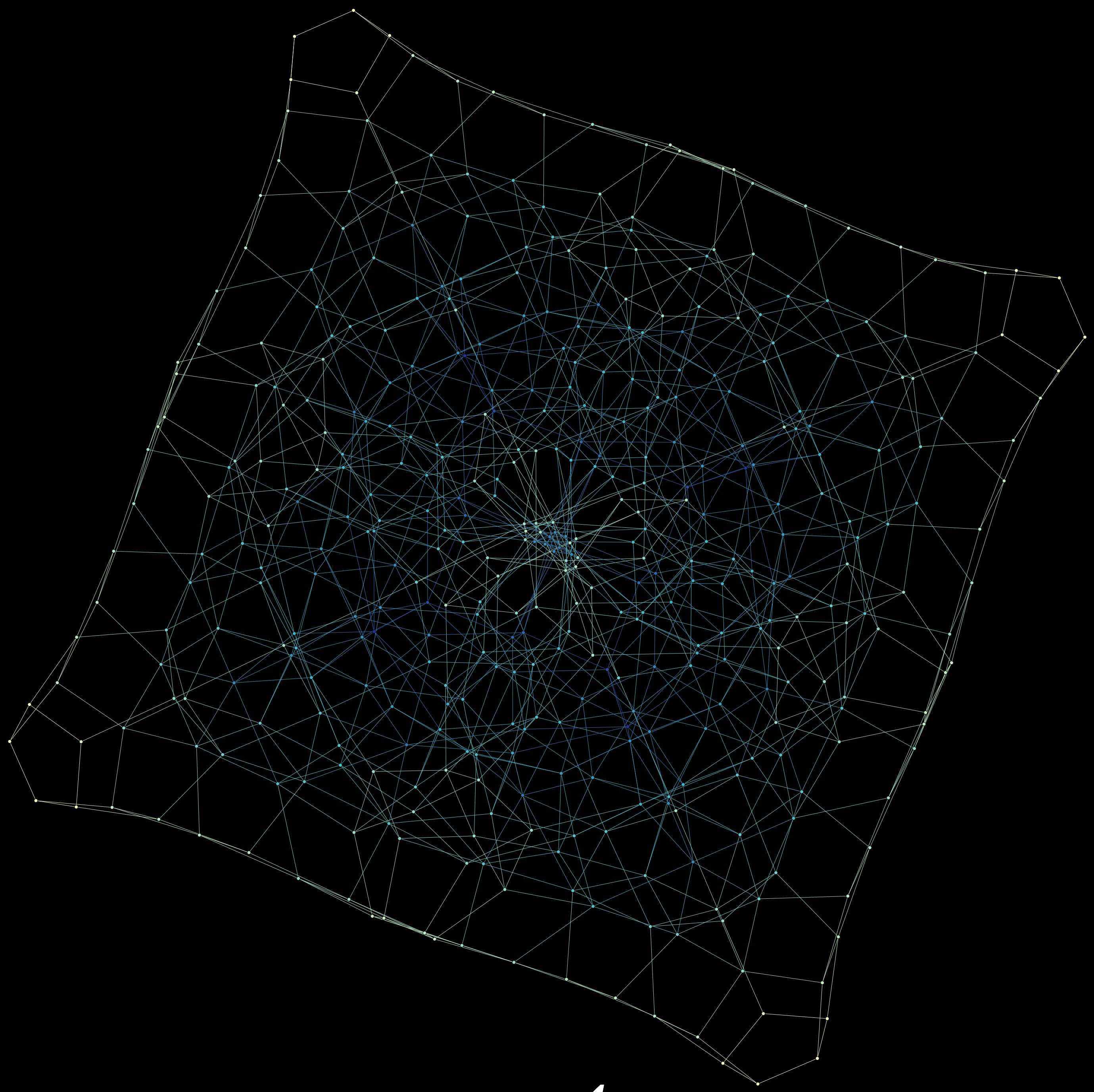


Theorem (P.-Pournin)

The diameter of the flip graph of a torus with $n>0$ boundary vertices is somewhere between $5n/2$ and $23n/8$.

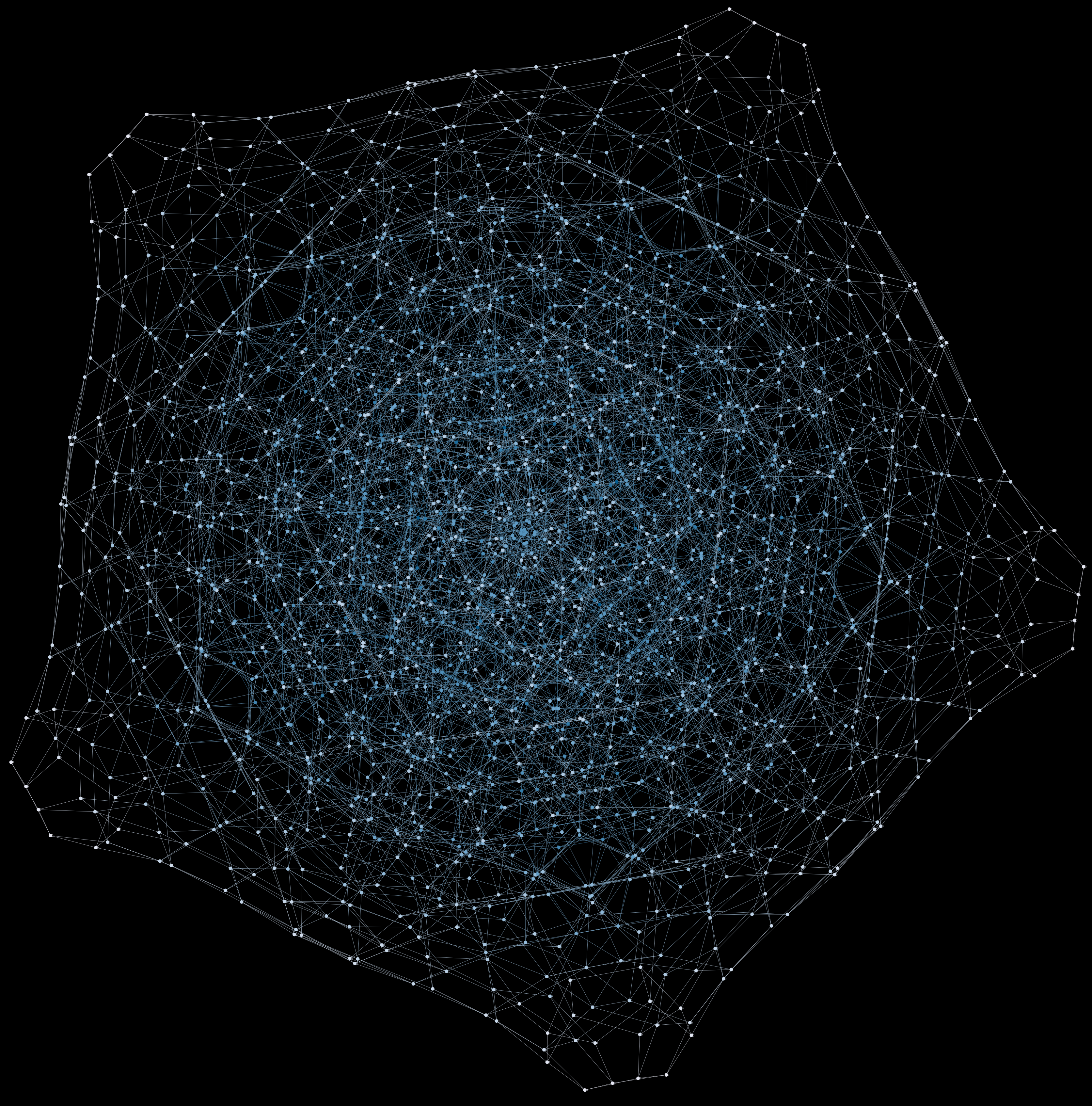


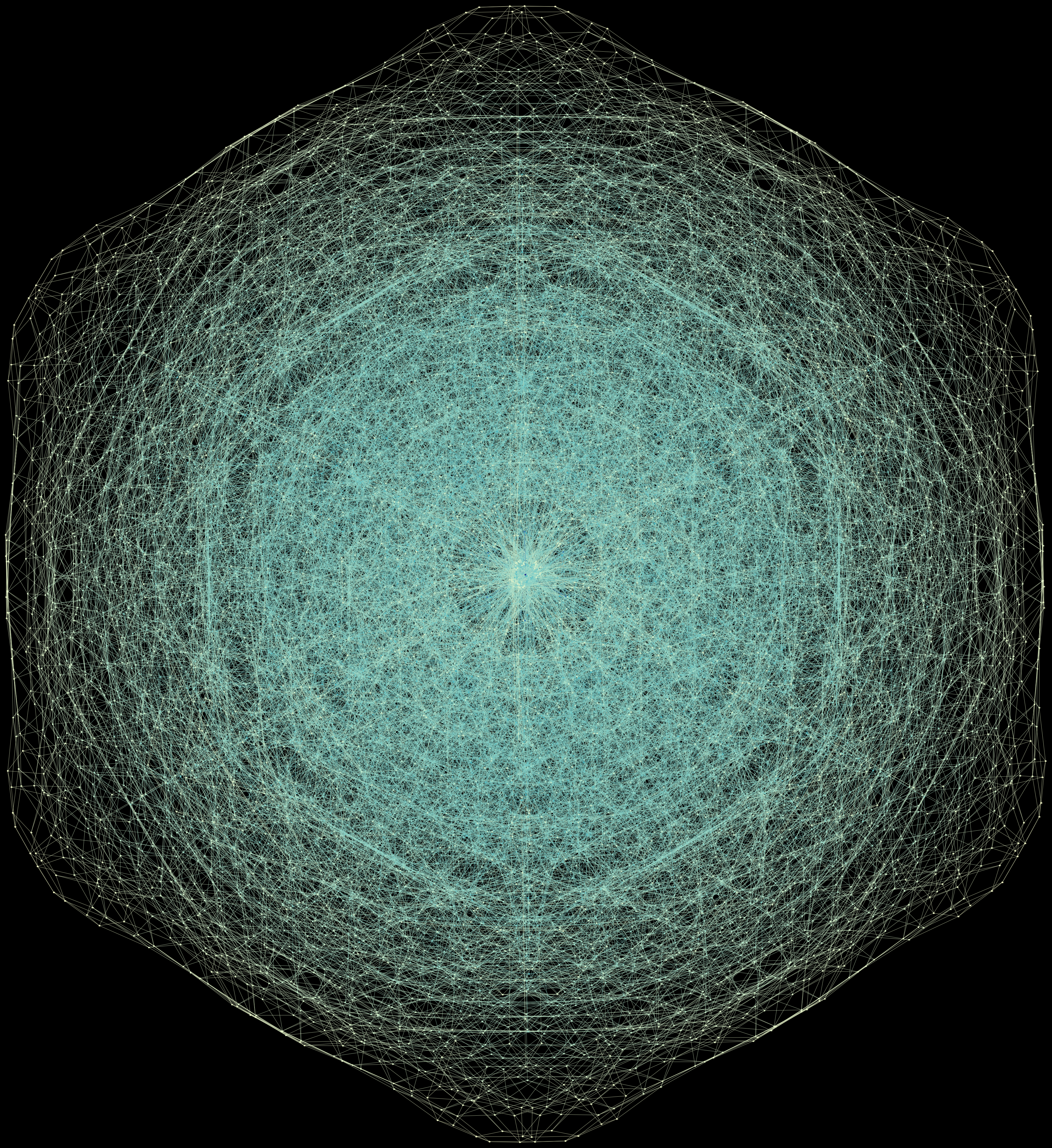
n=3

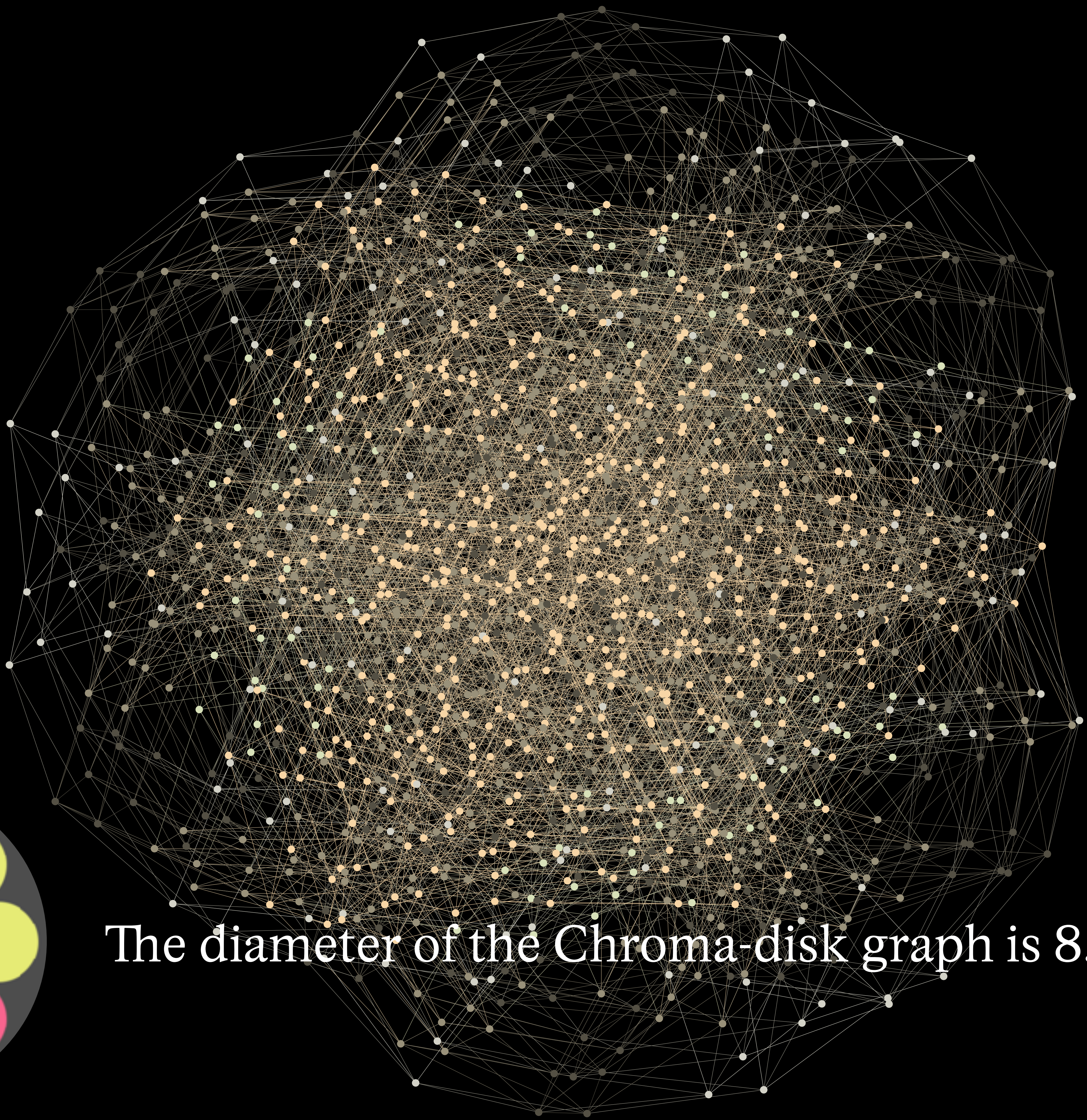


n=4

Torus flip graphs

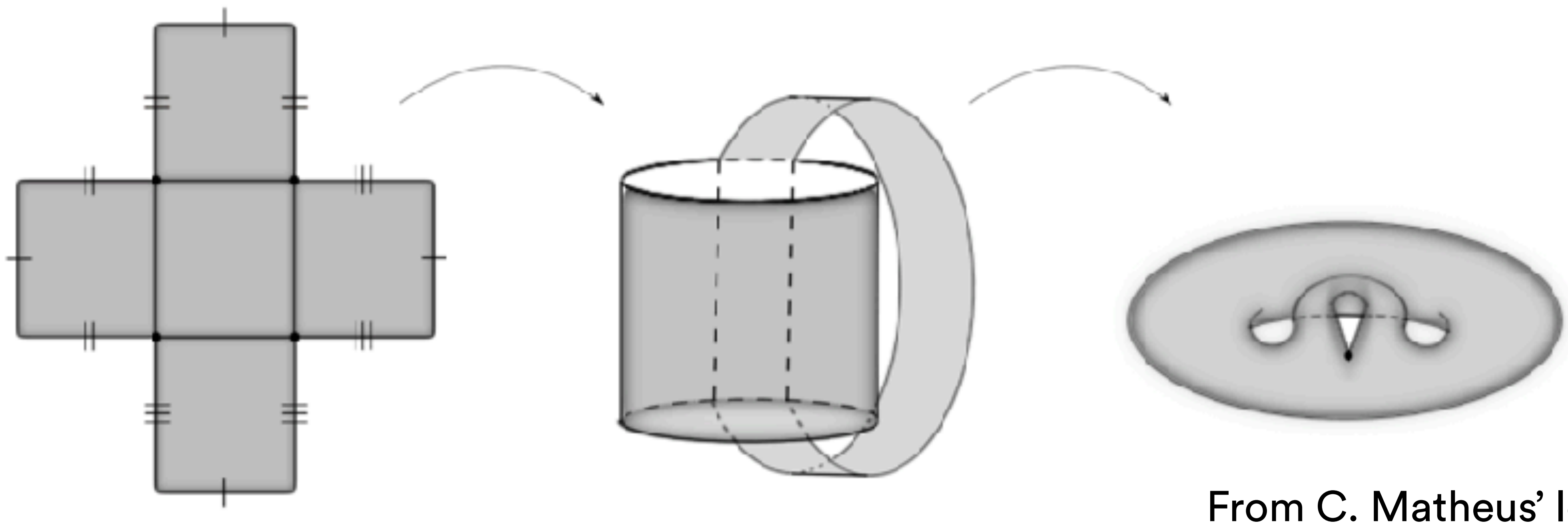




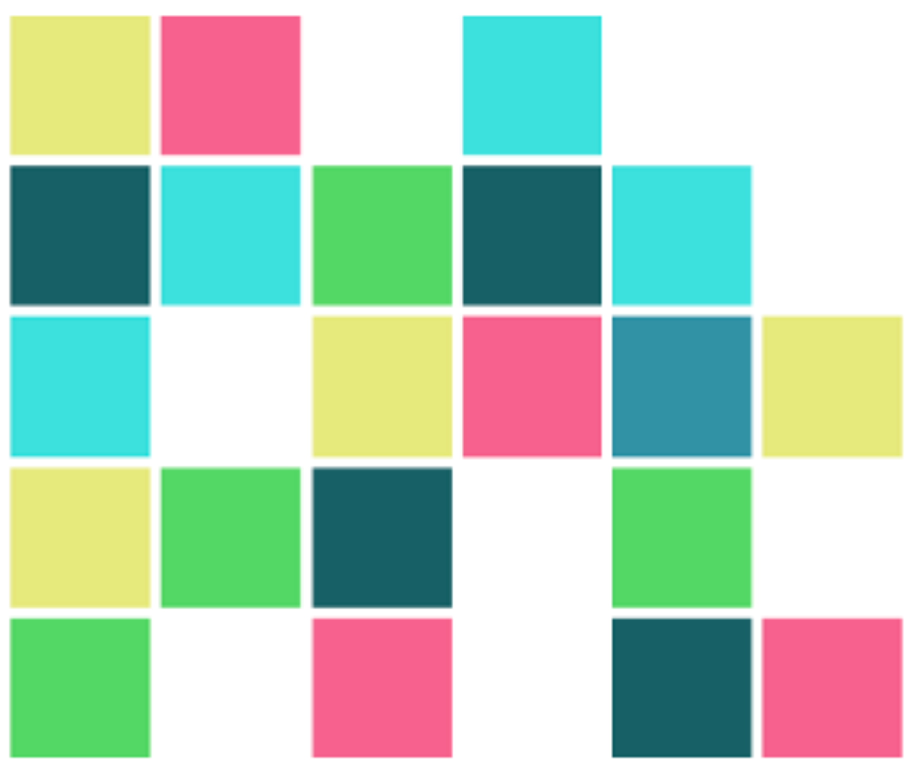


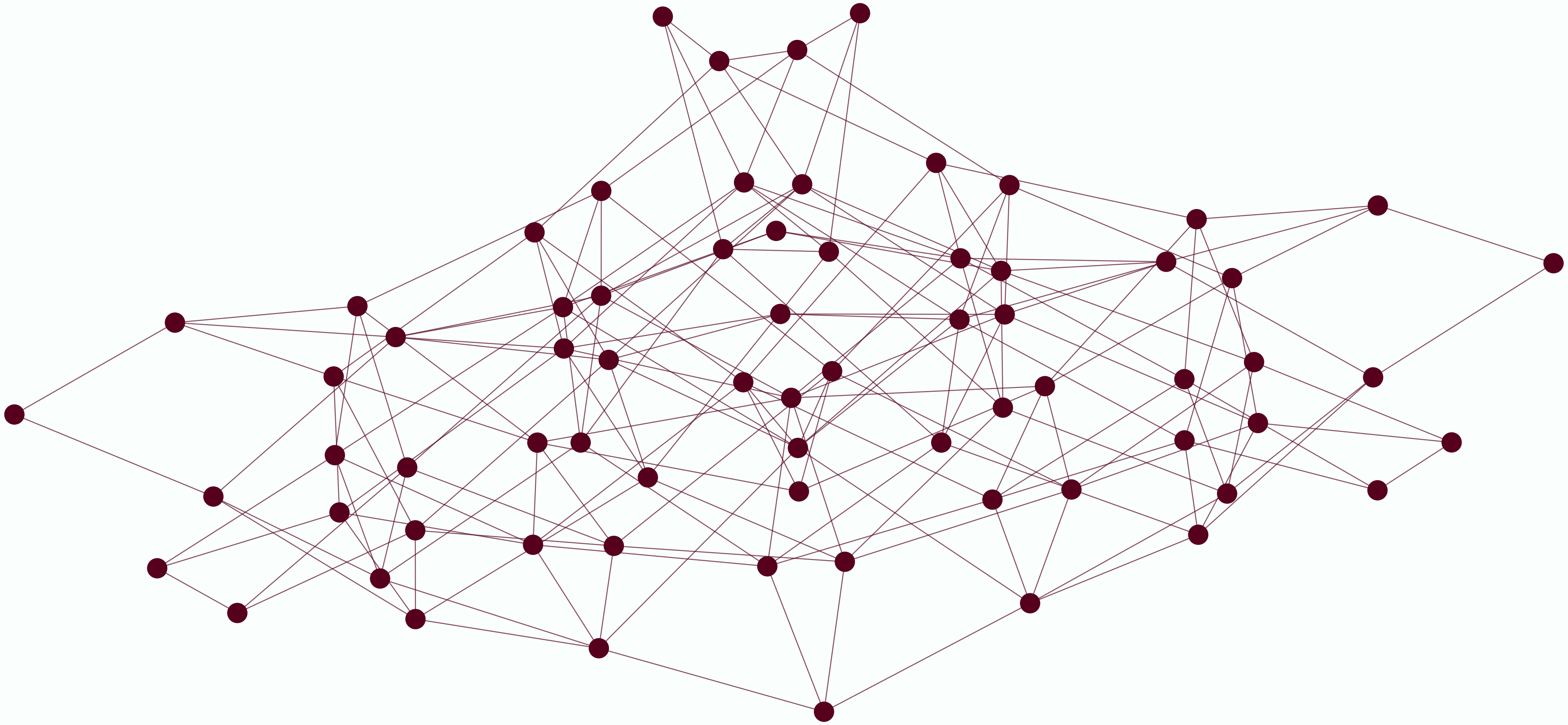
The diameter of the Chroma-disk graph is 8.

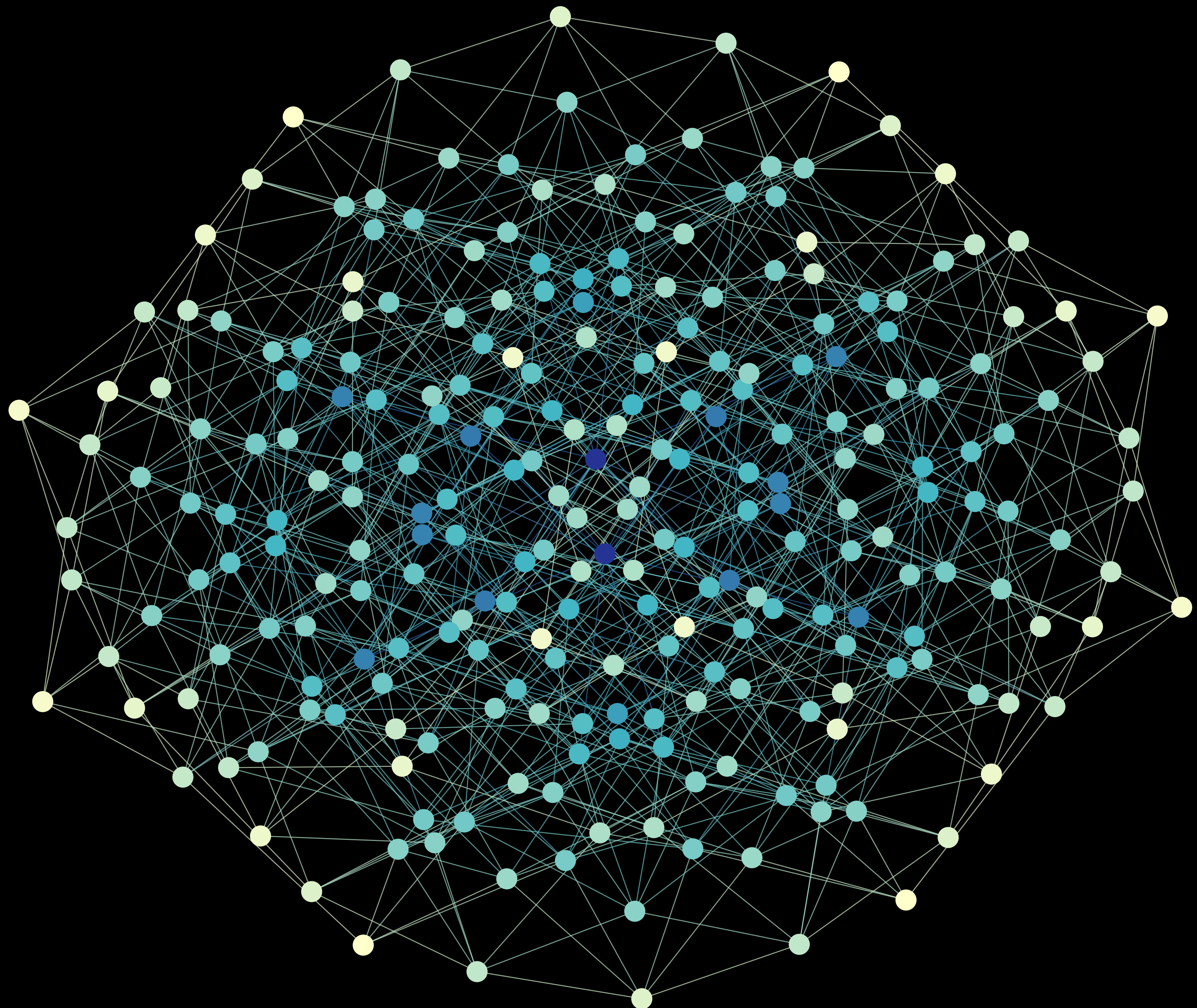
Puzzle graphs for square tiled translation surfaces
With Mario Guttierez and Paul Turner

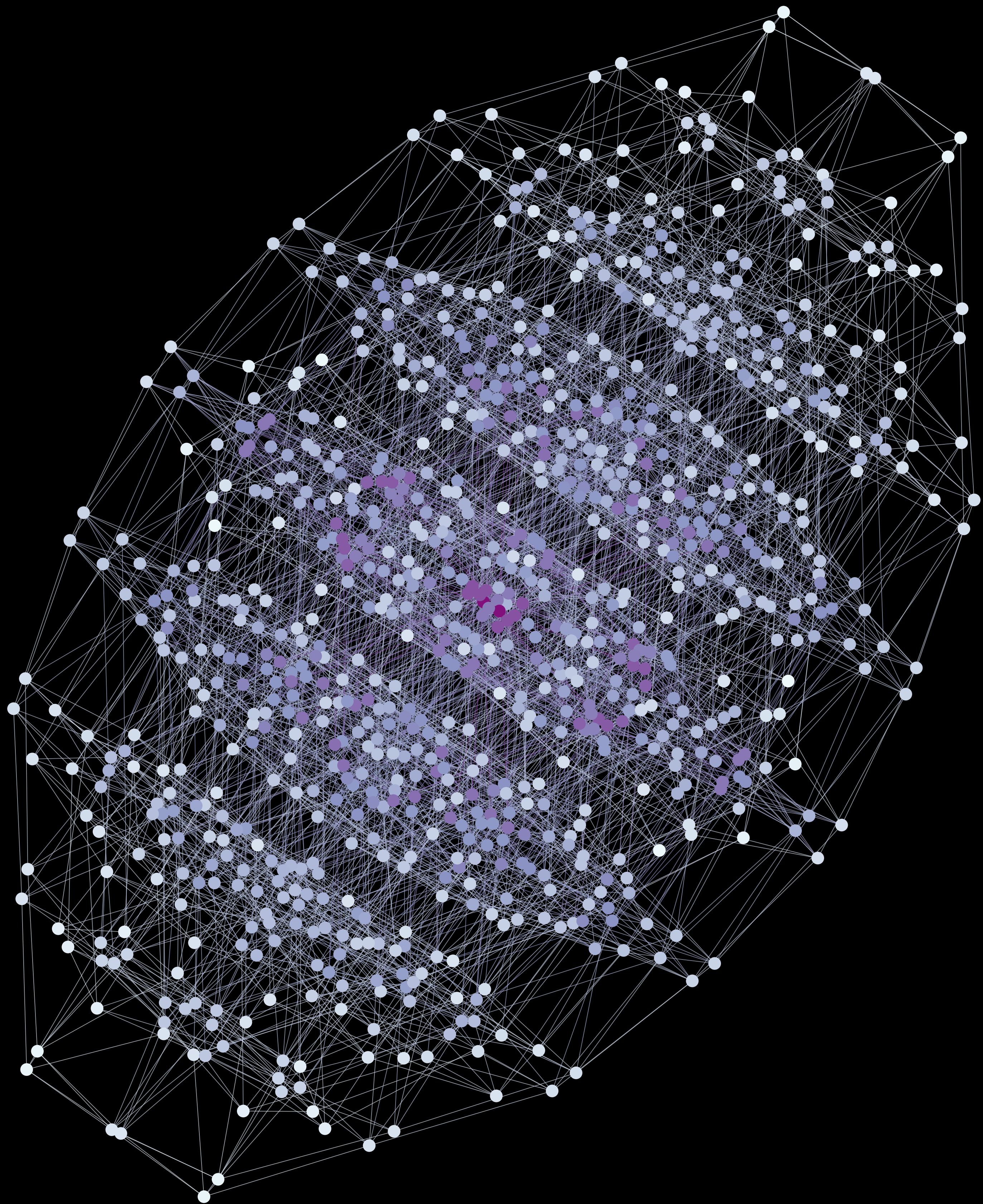


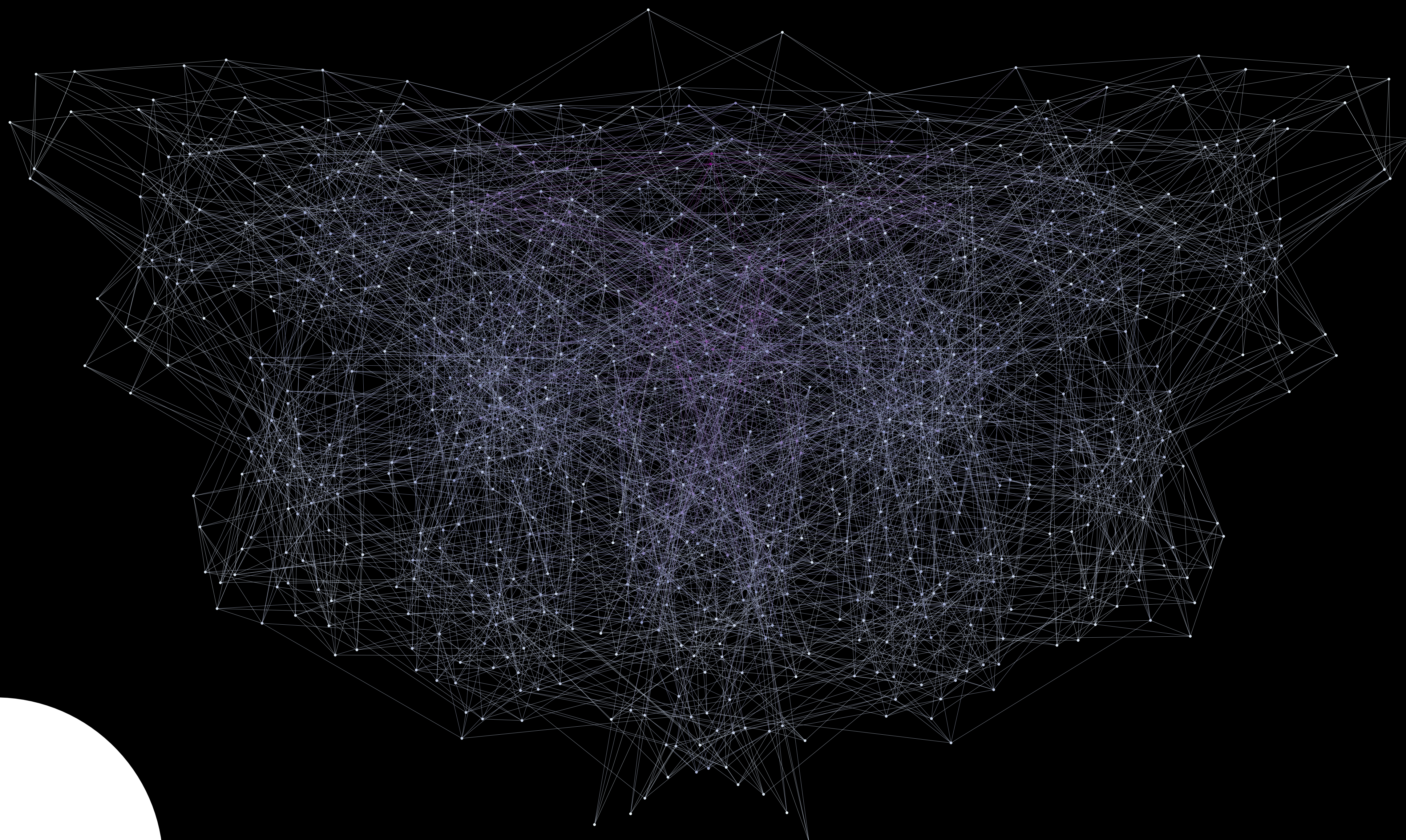
From C. Matheus' lecture notes

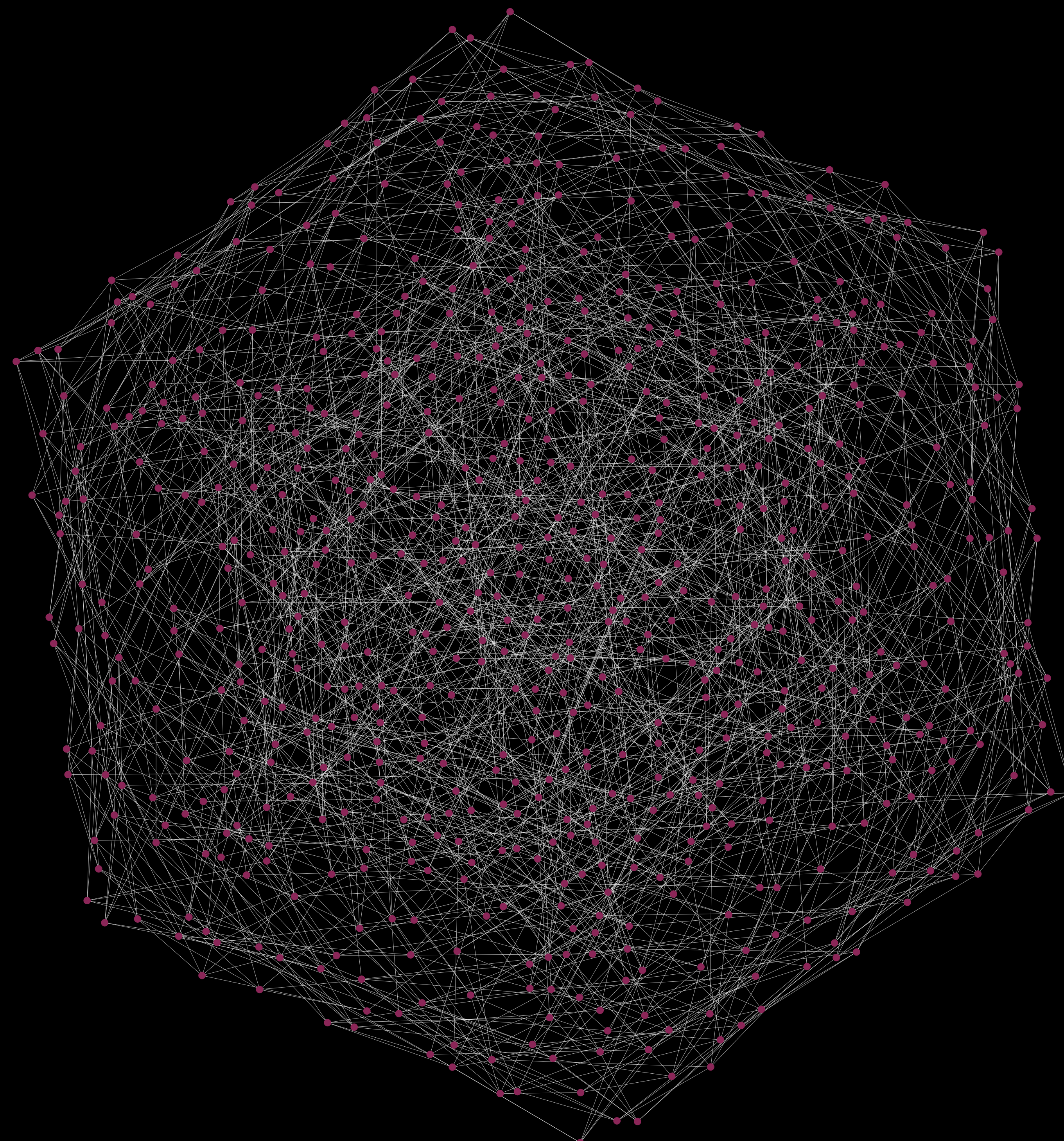


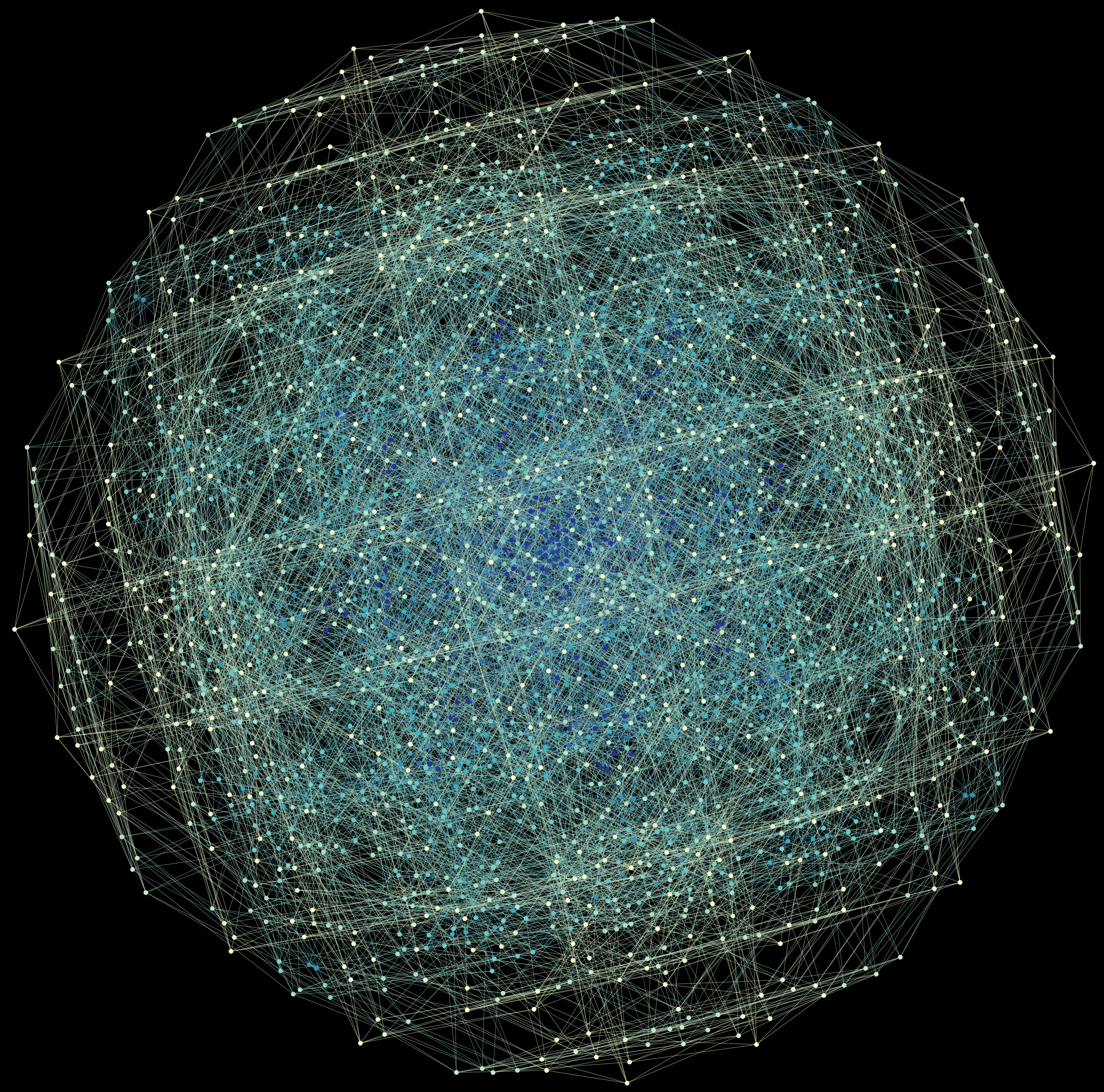


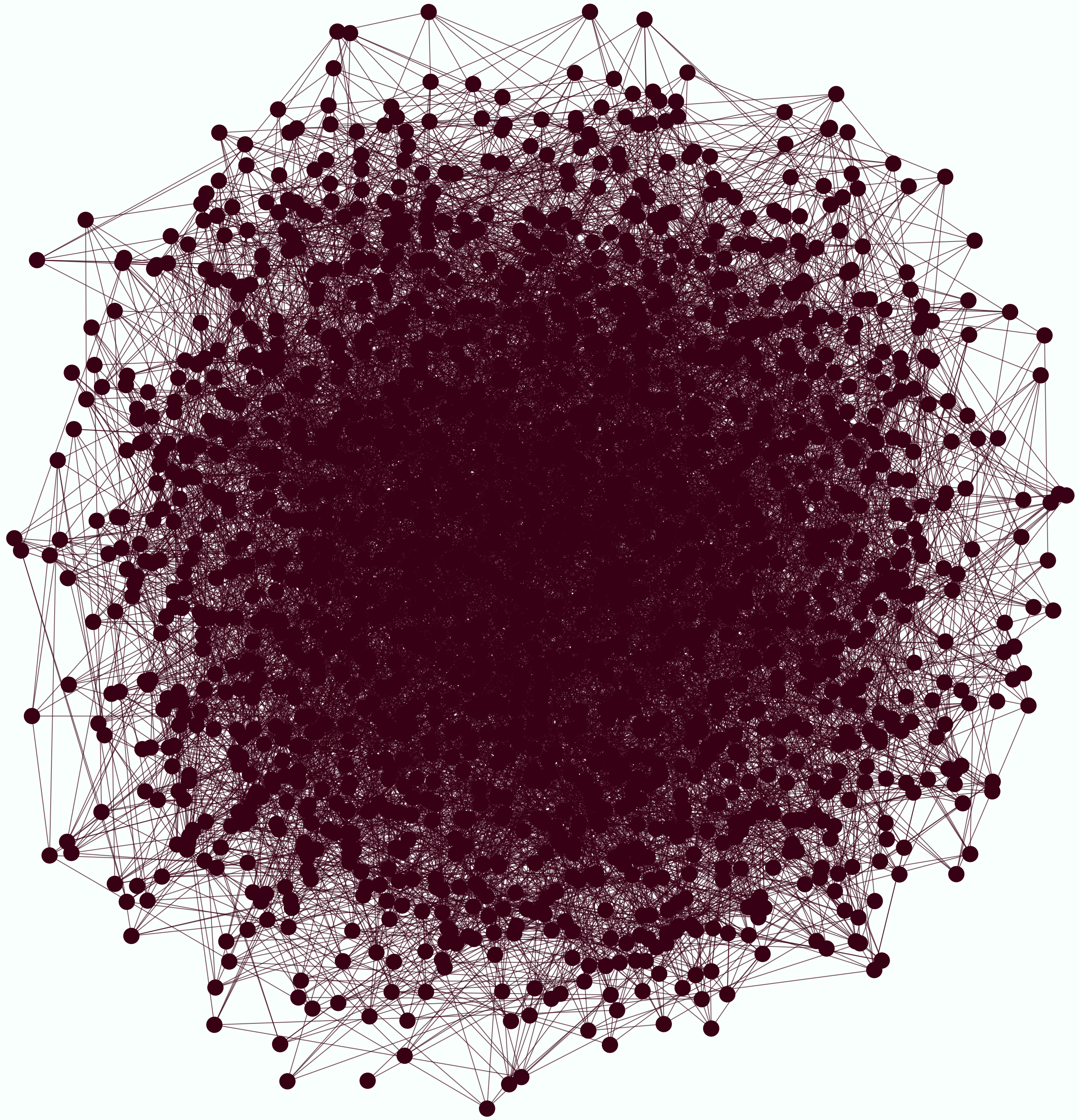


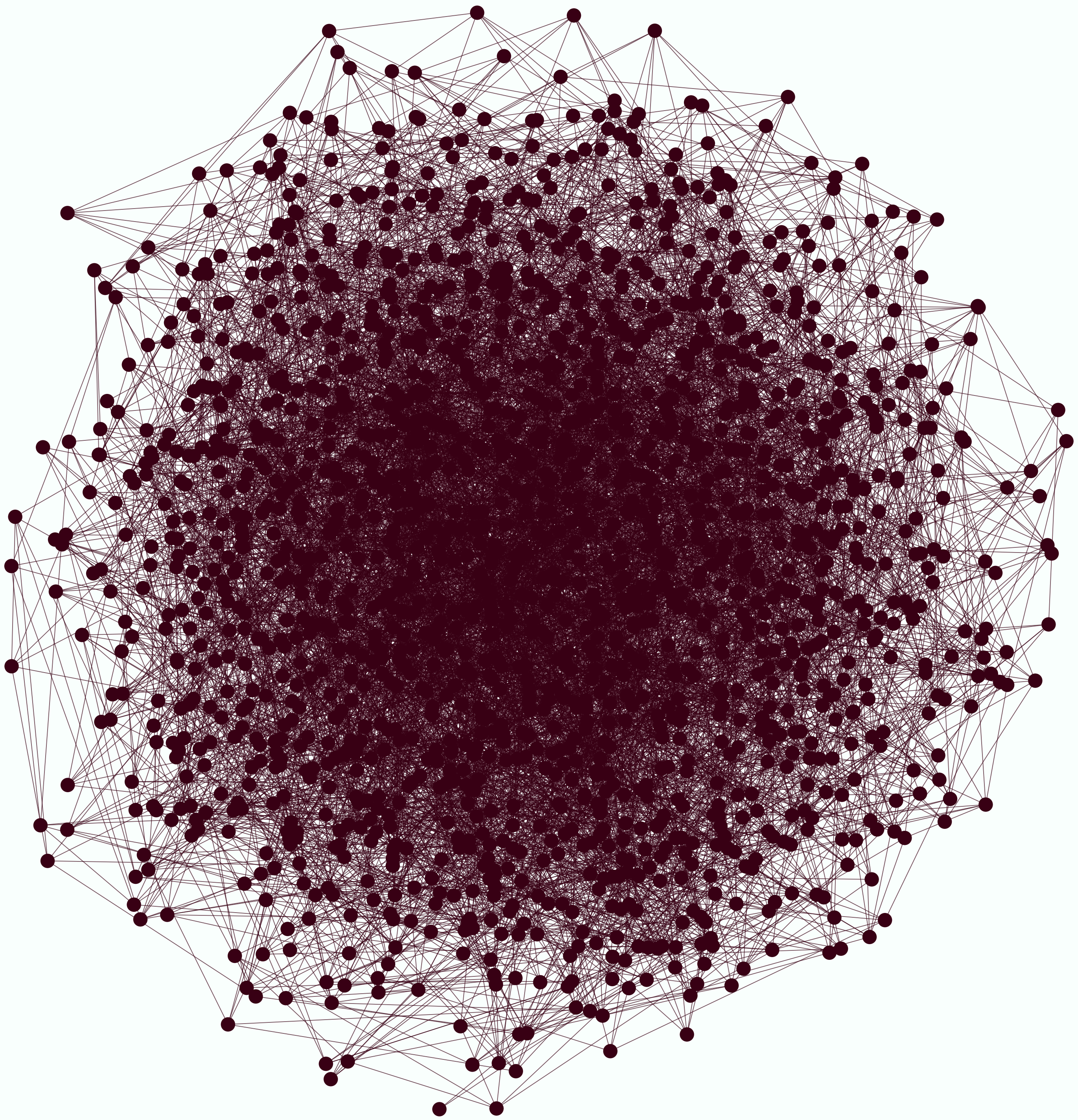


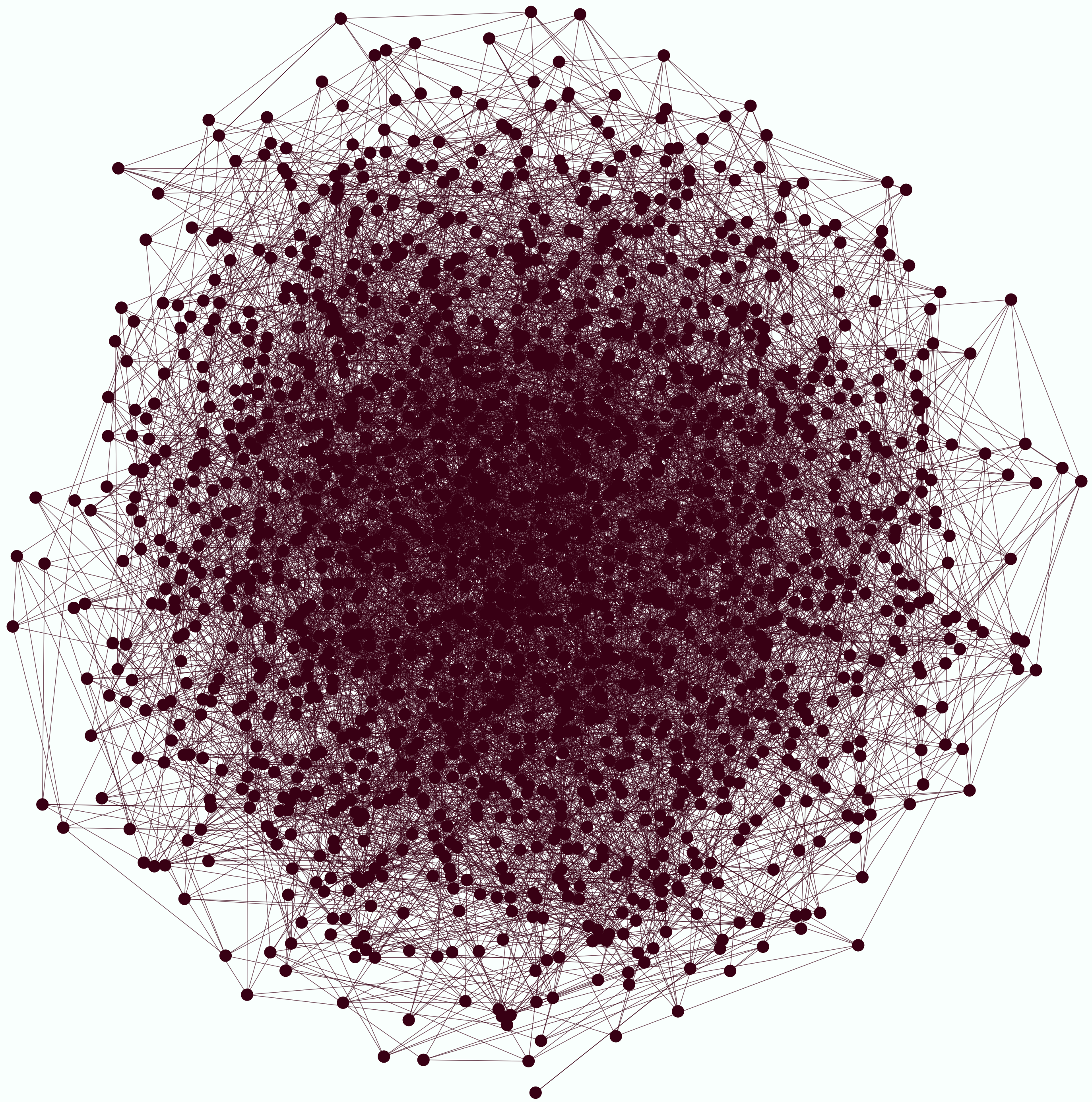


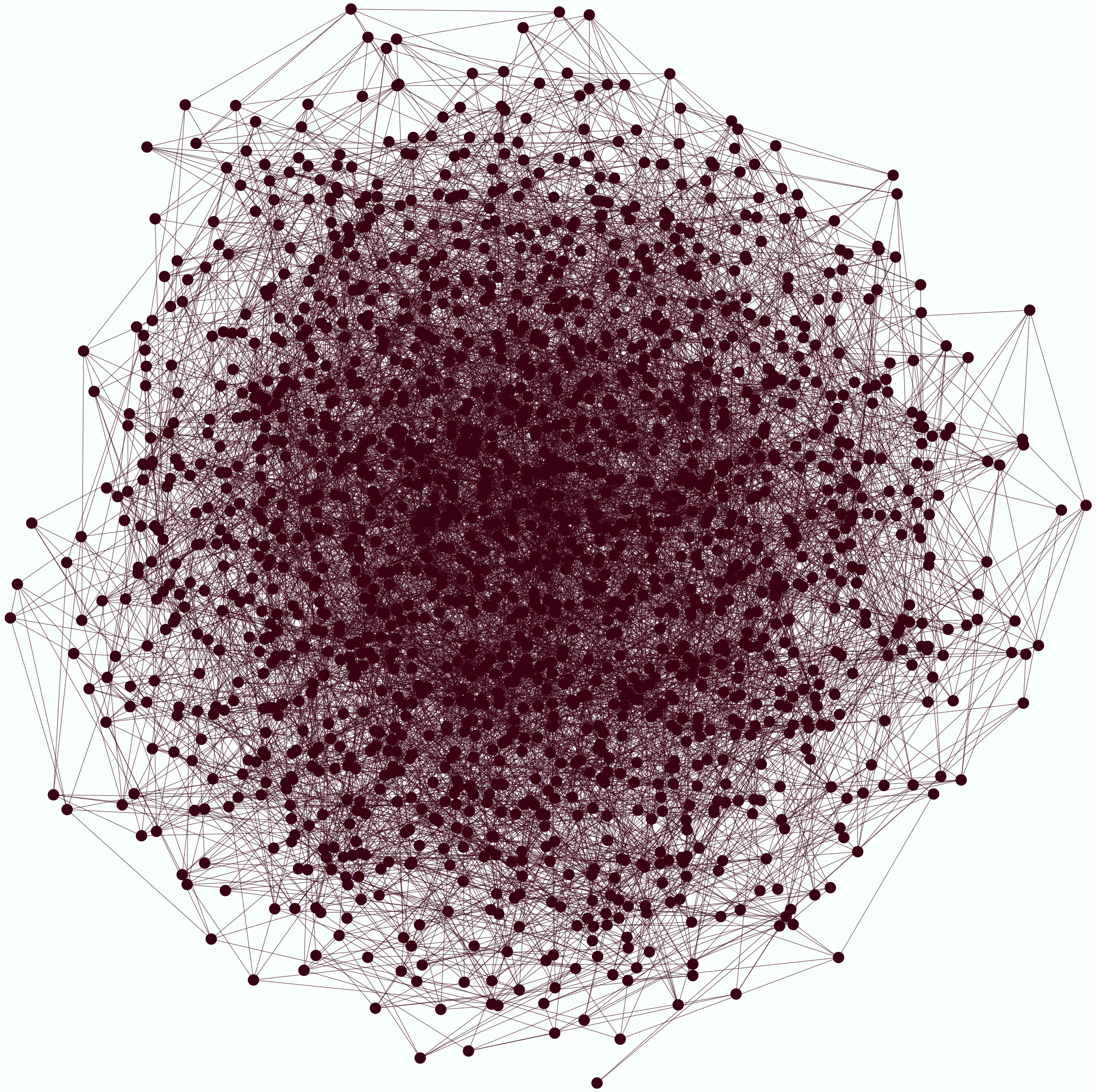


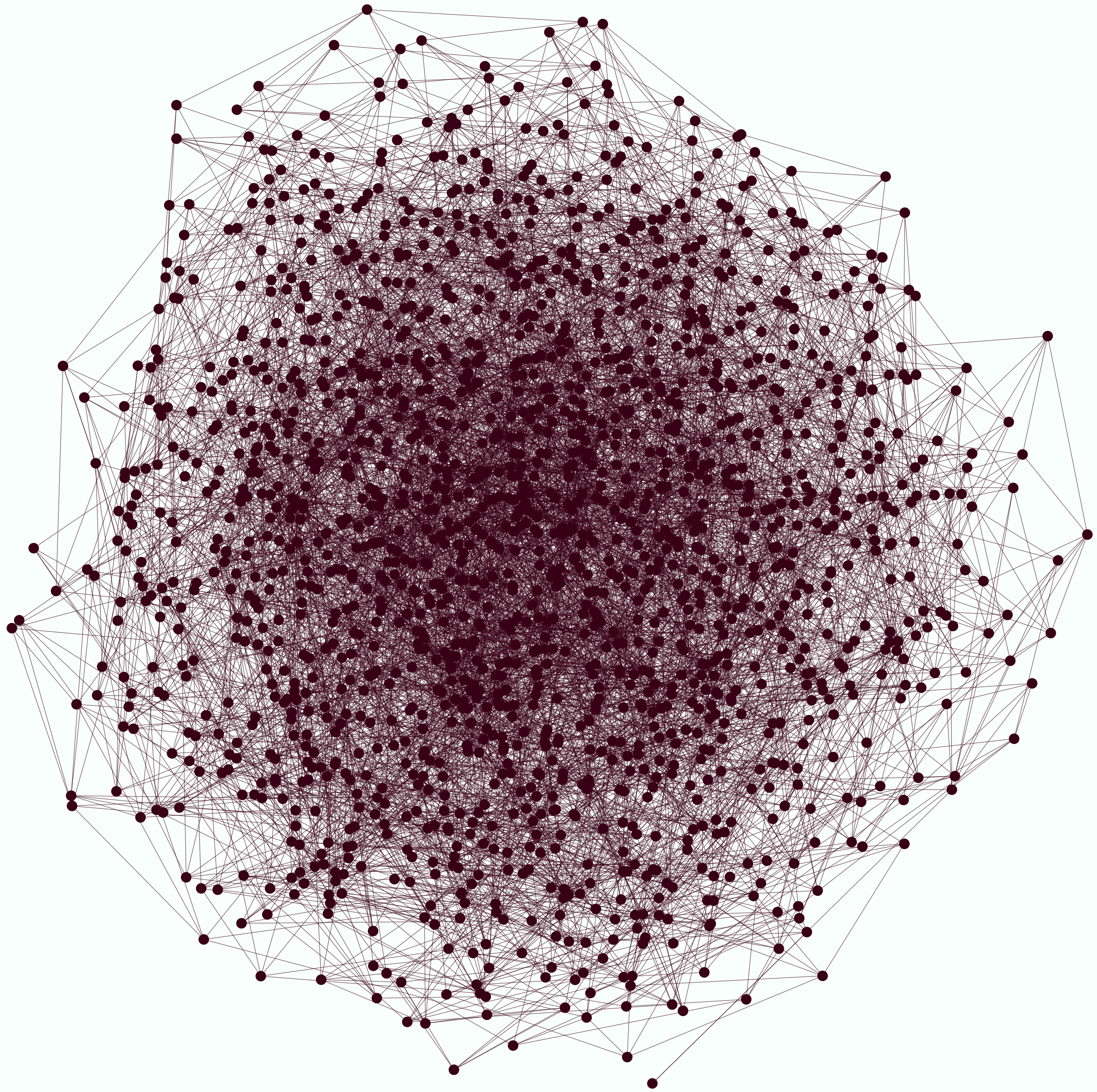


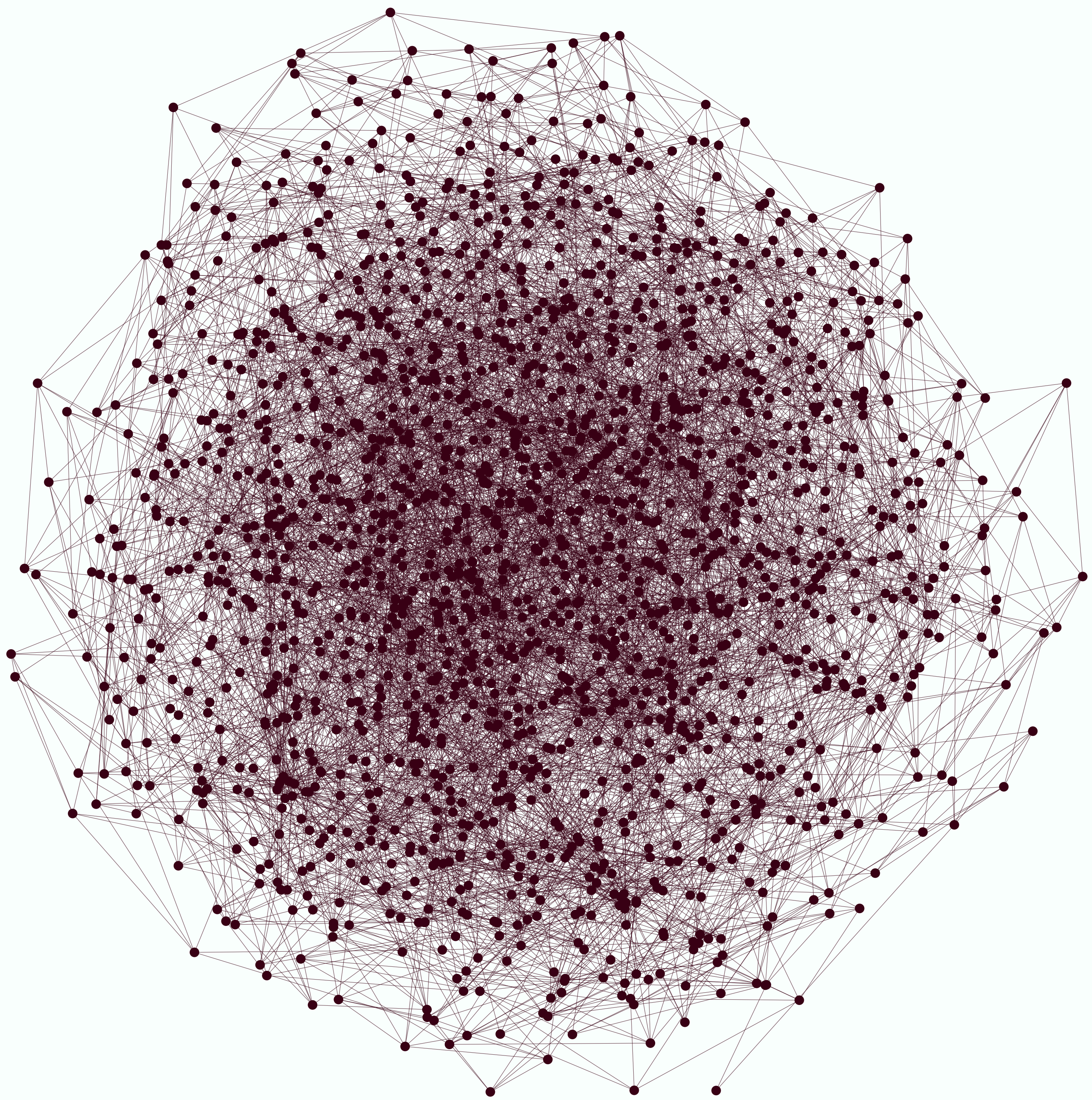


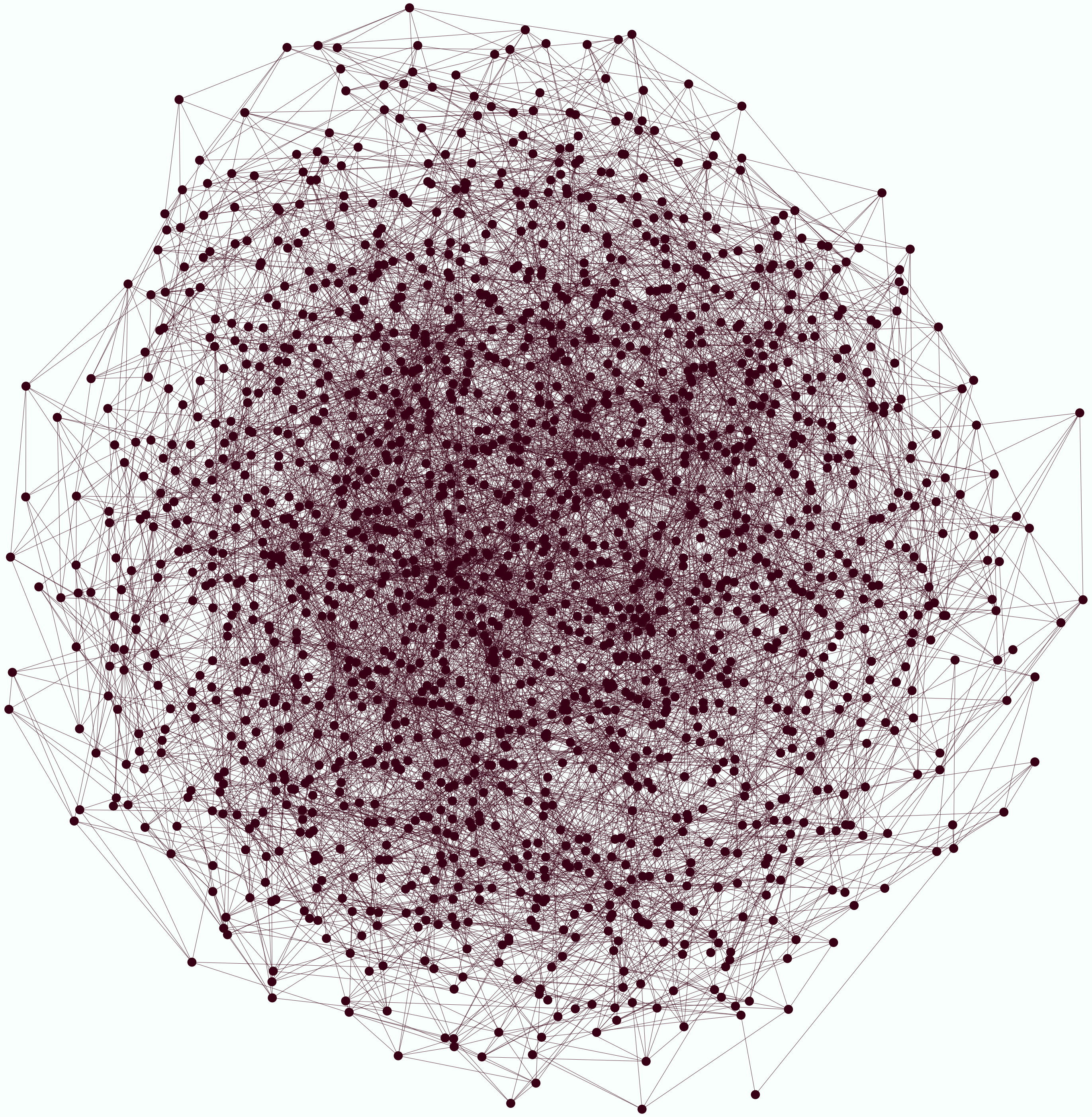


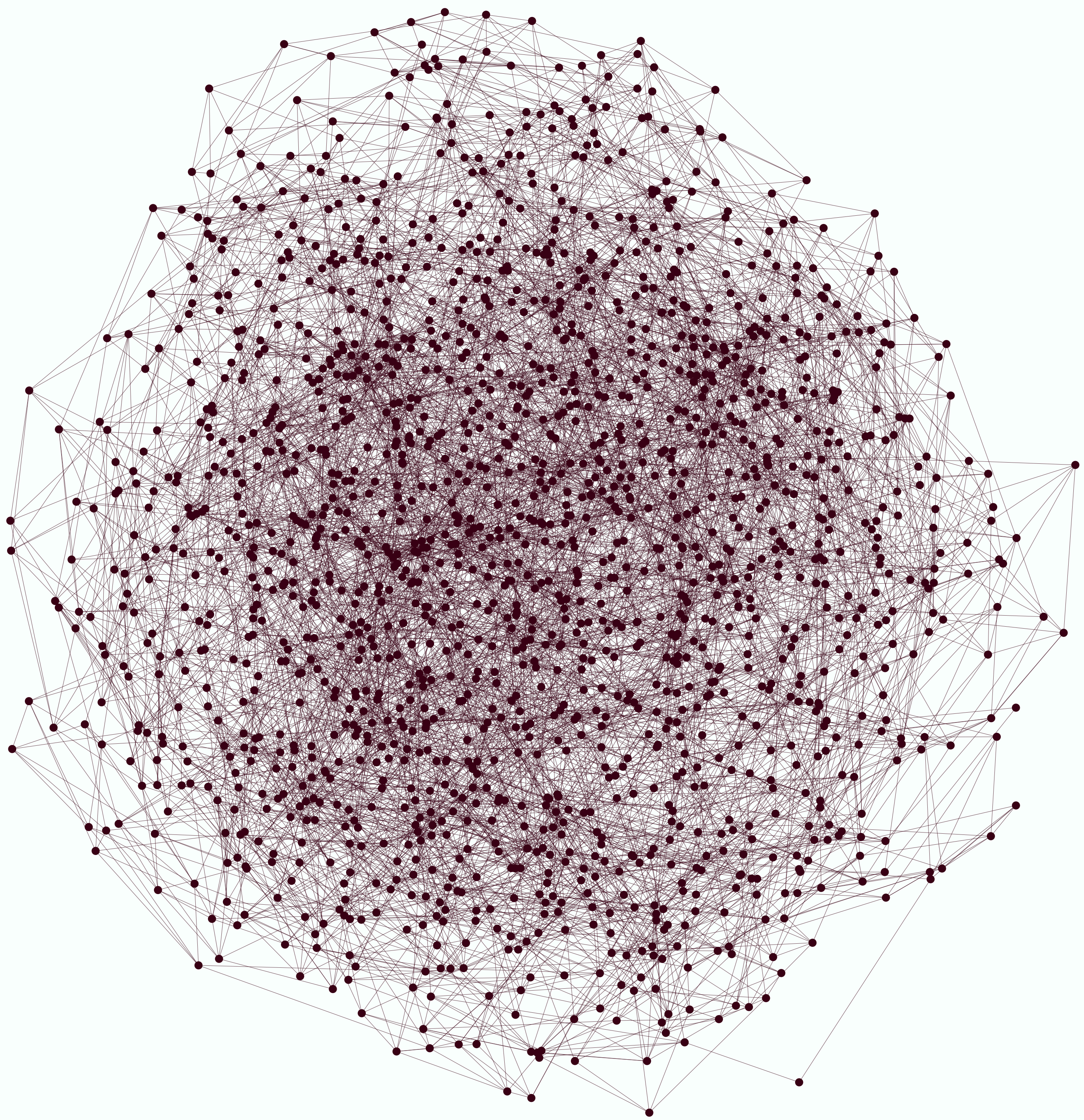


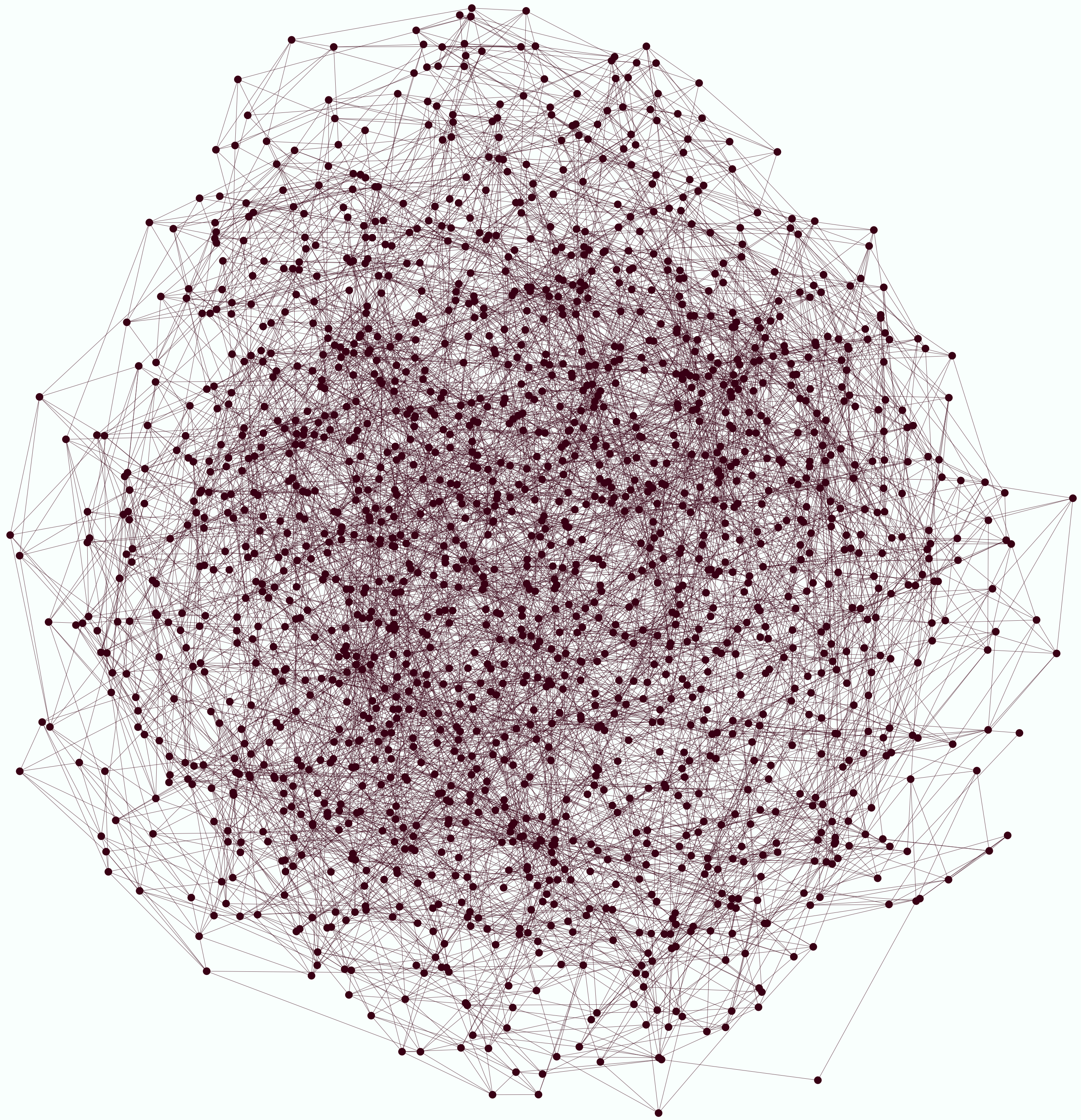


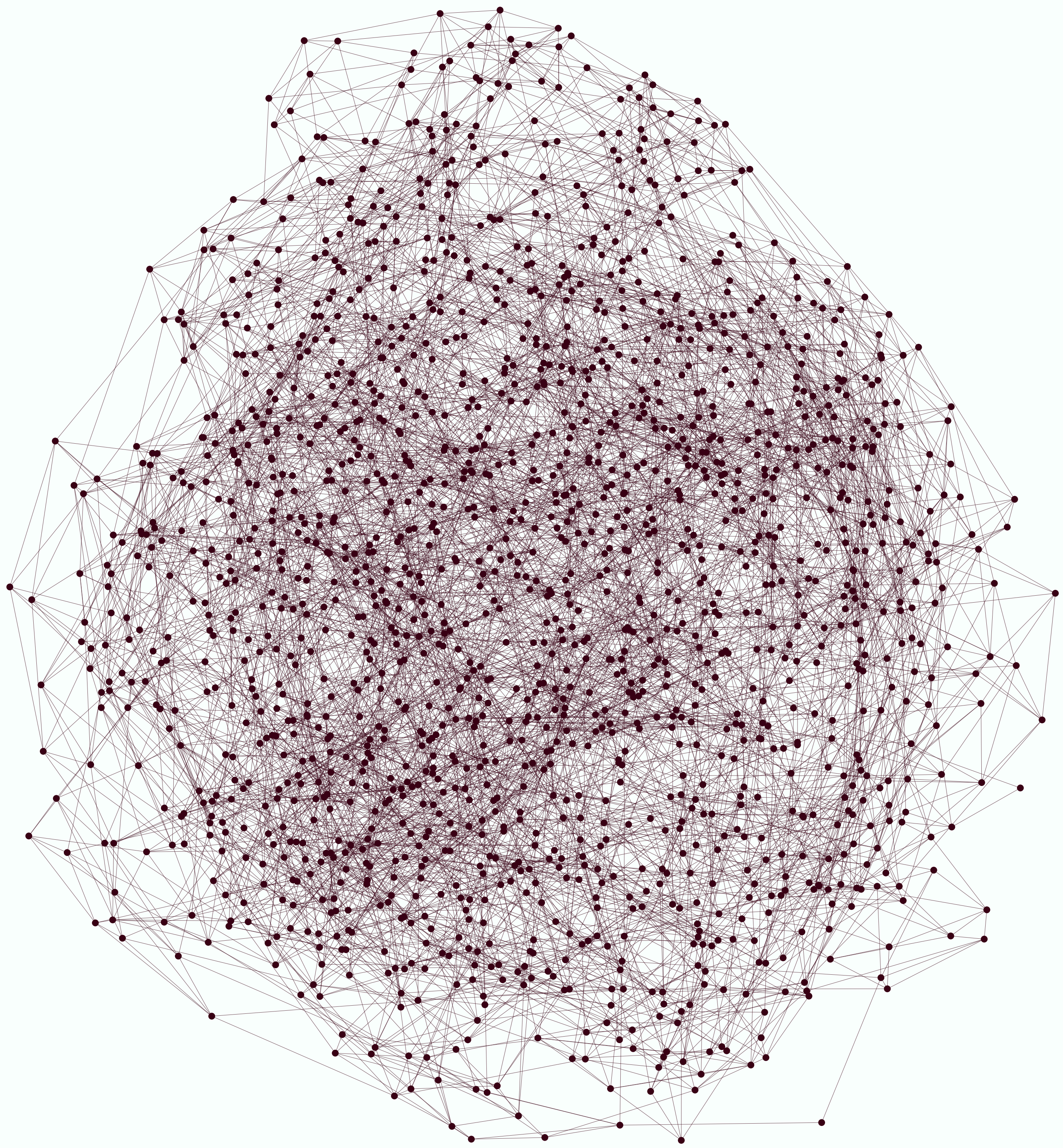


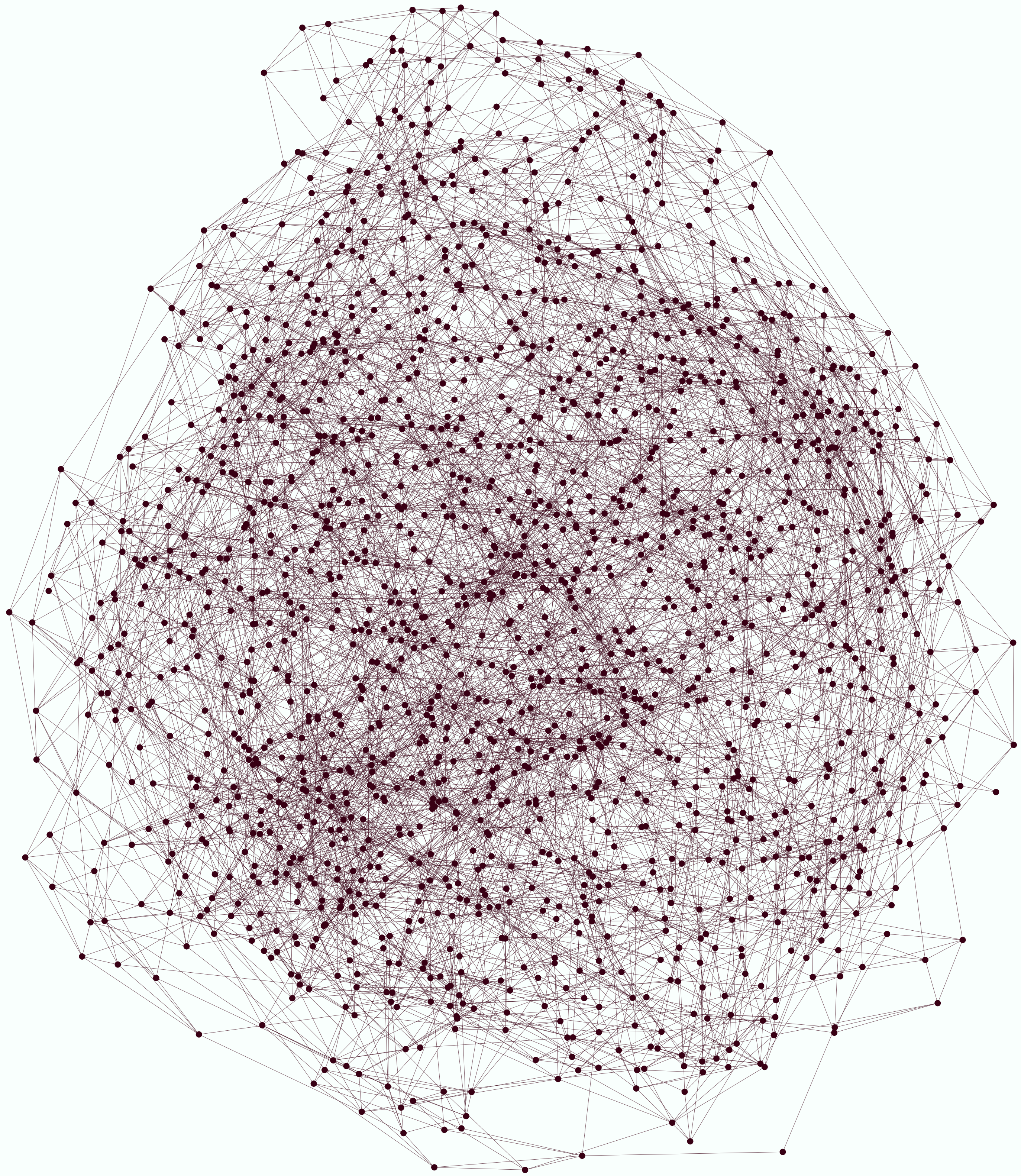


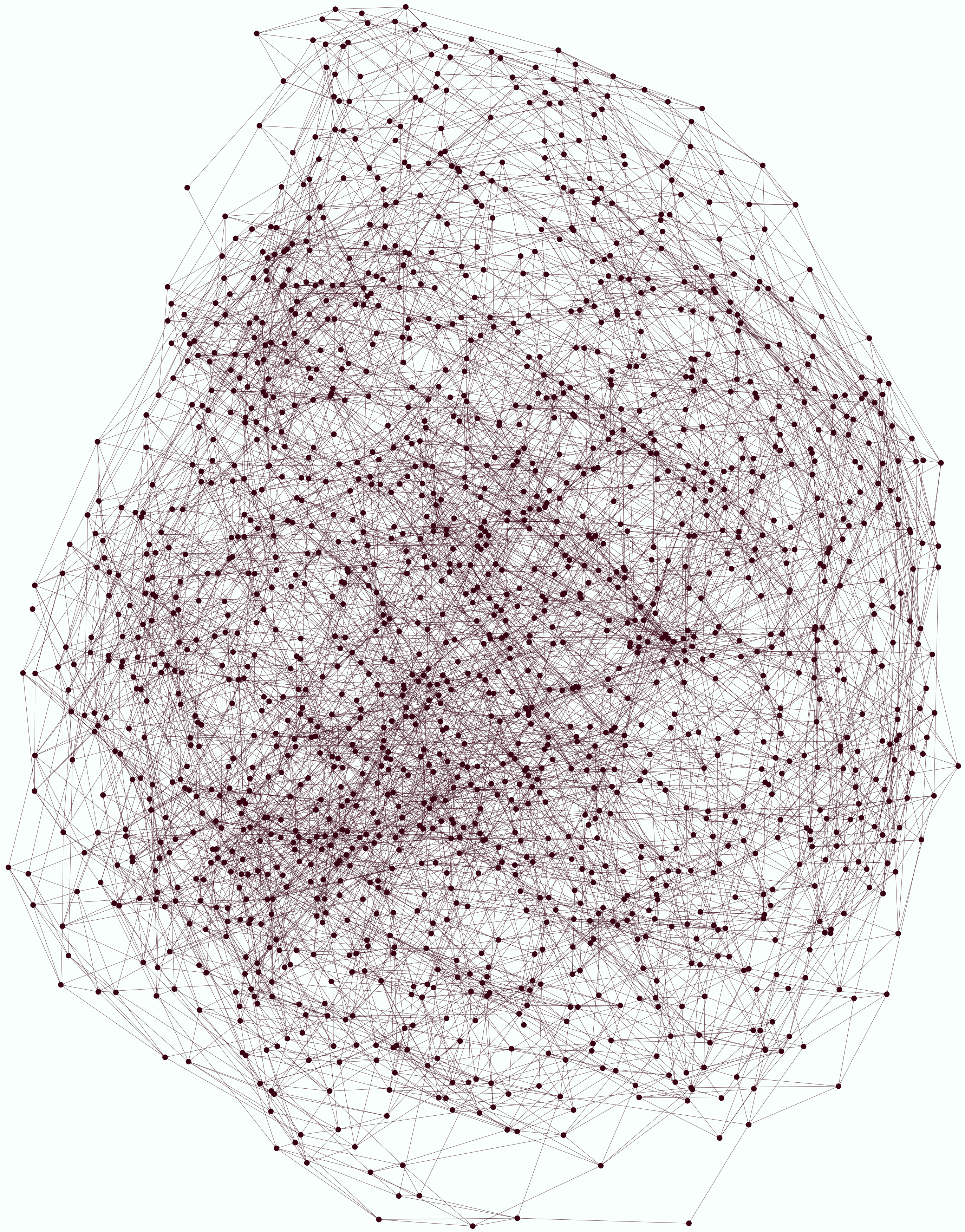


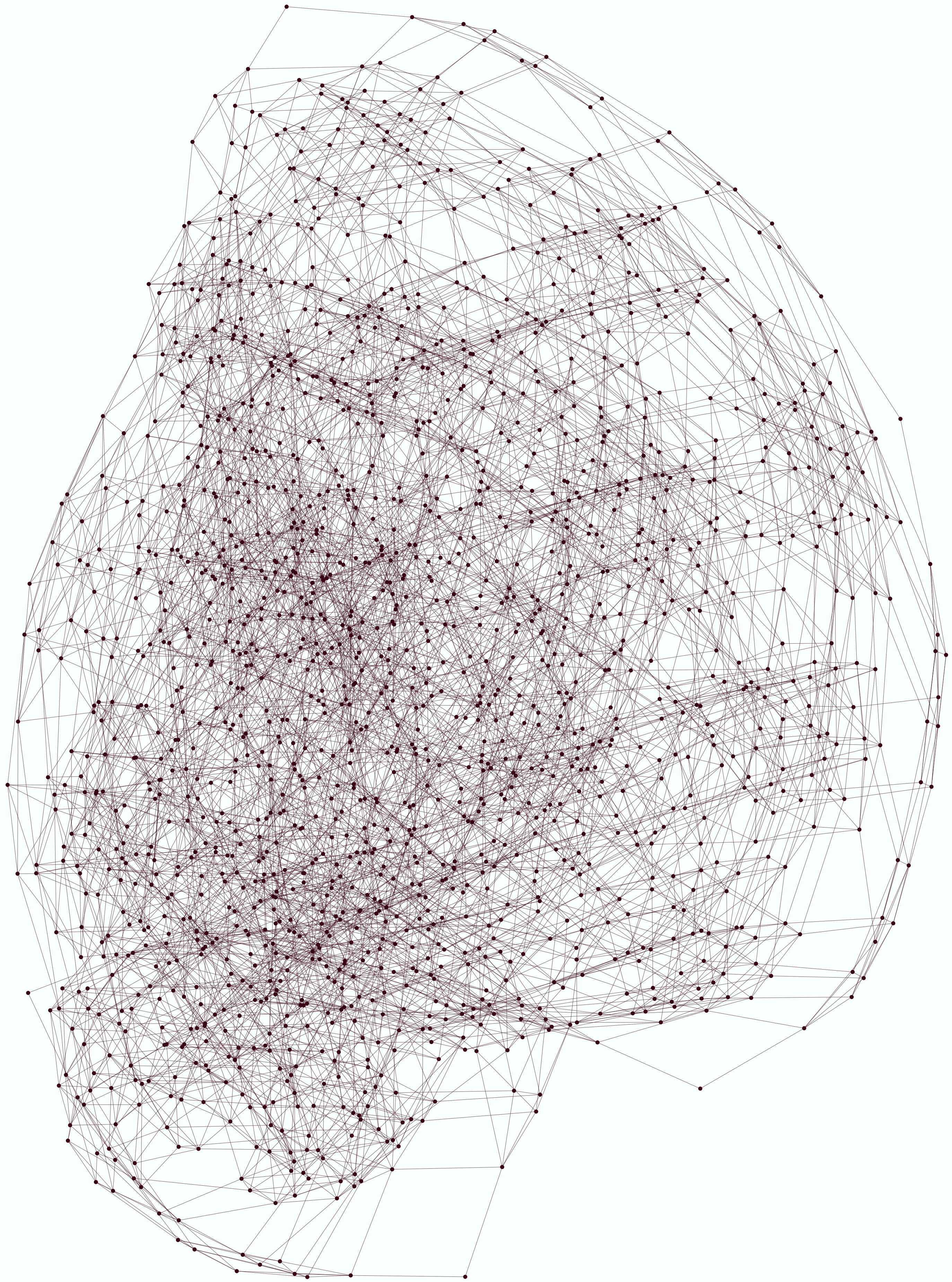


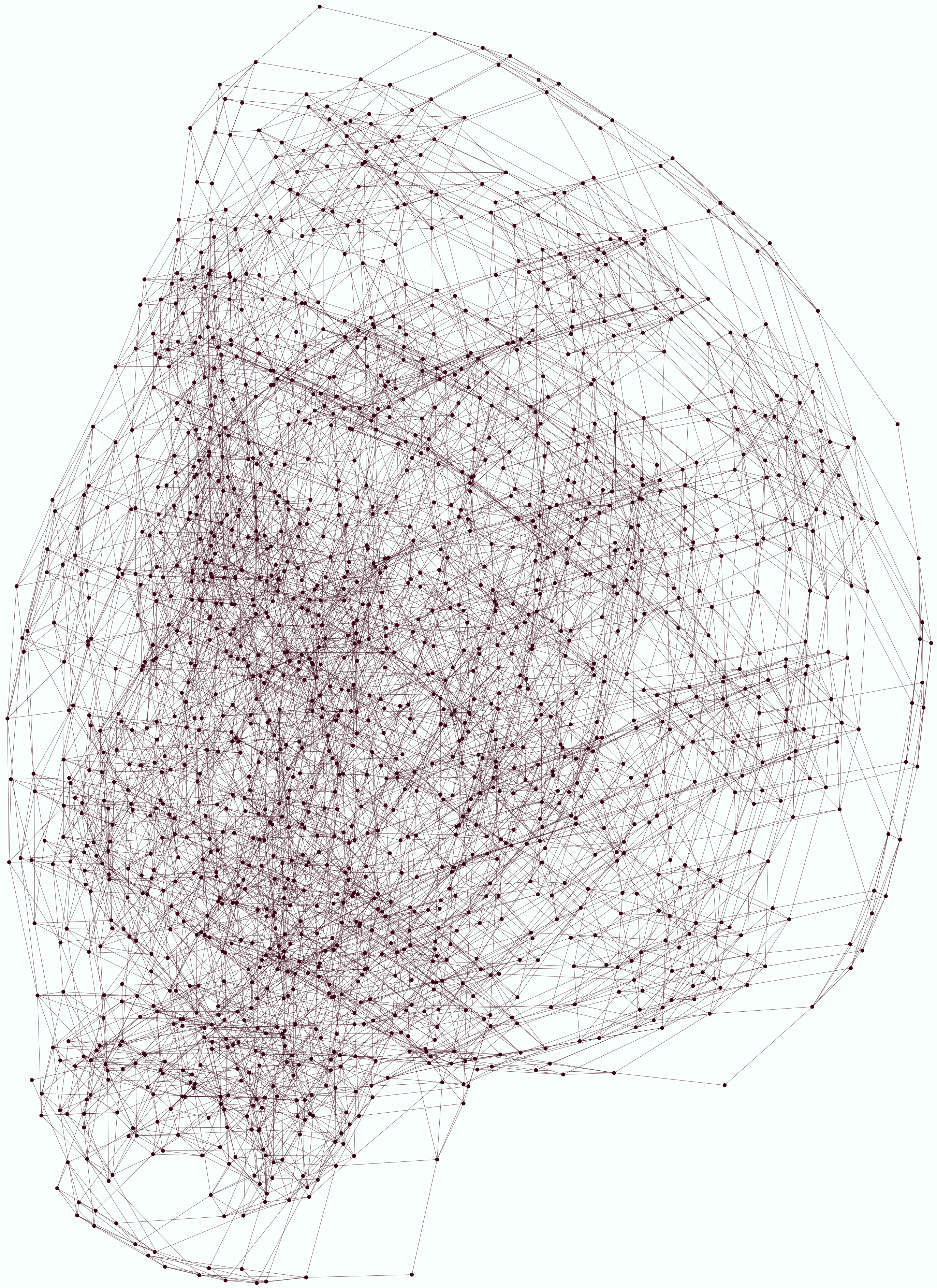


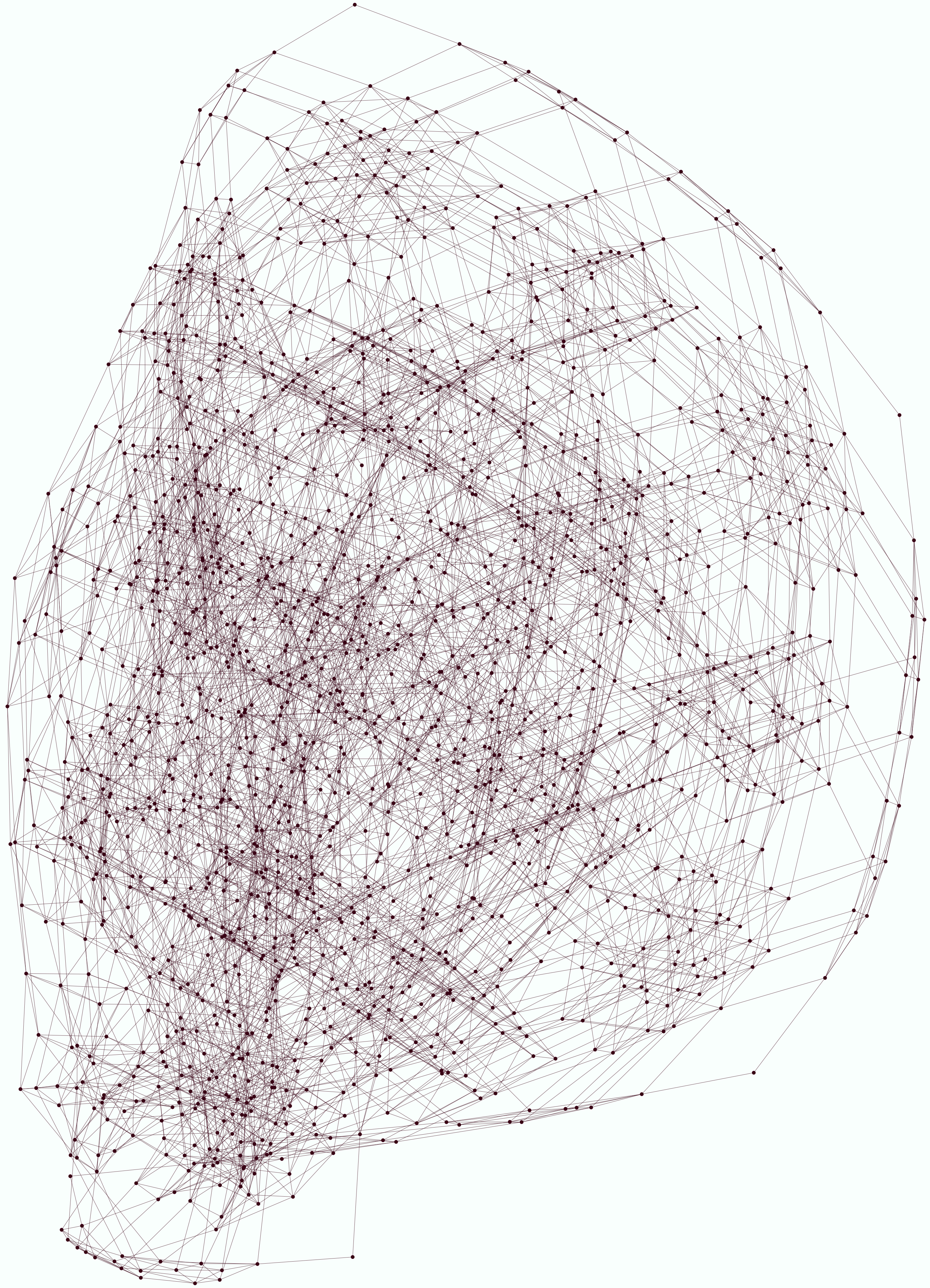


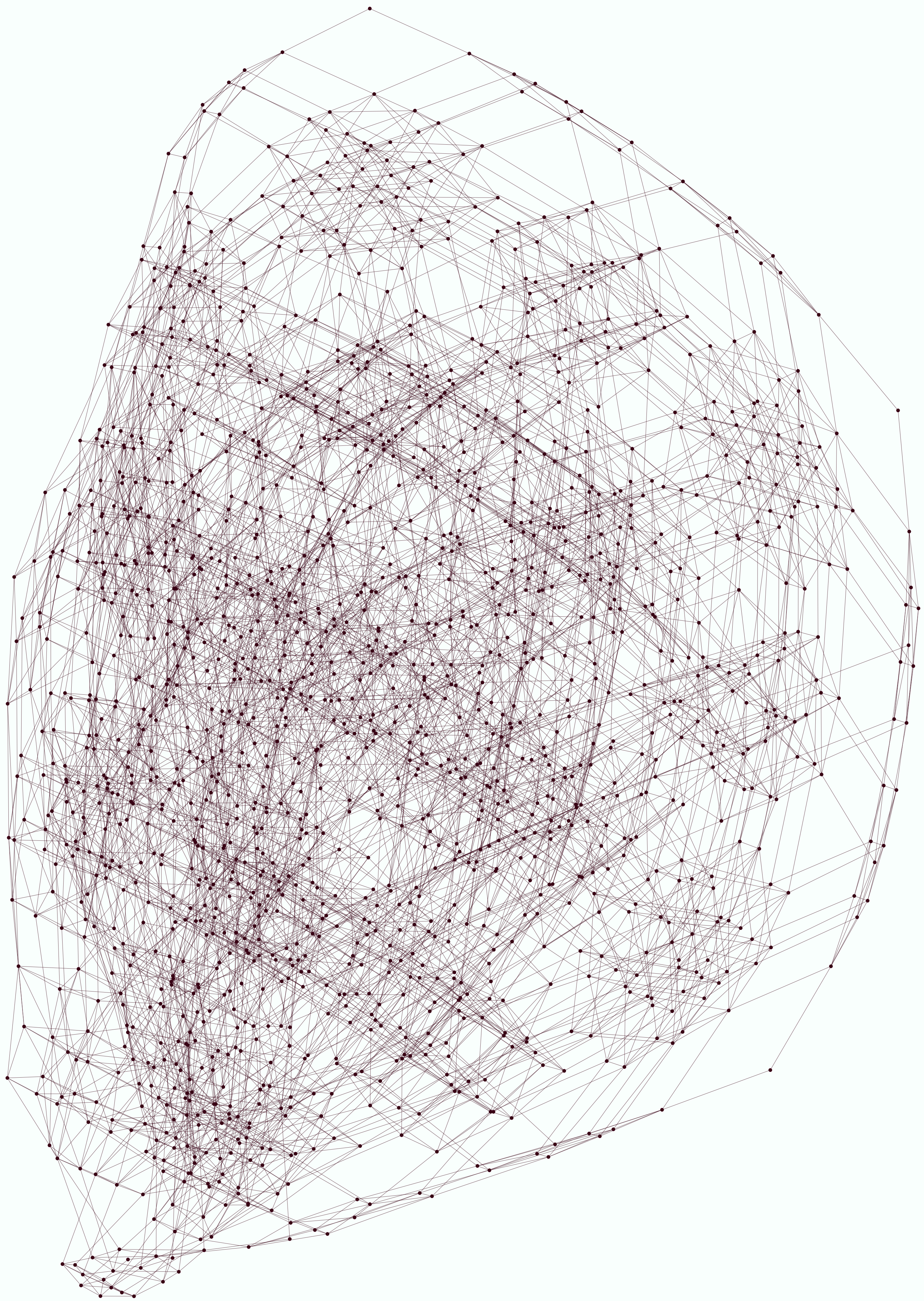


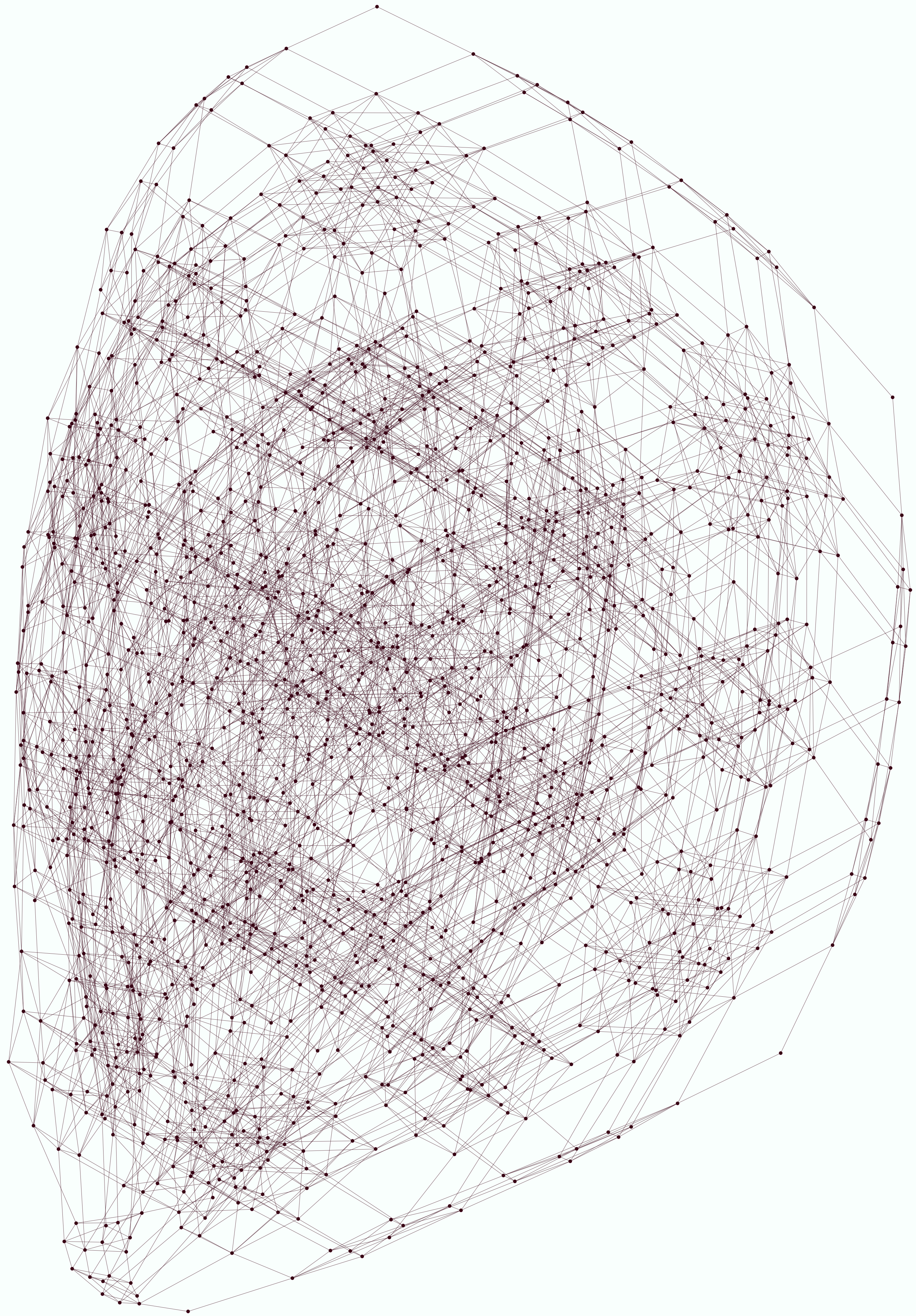


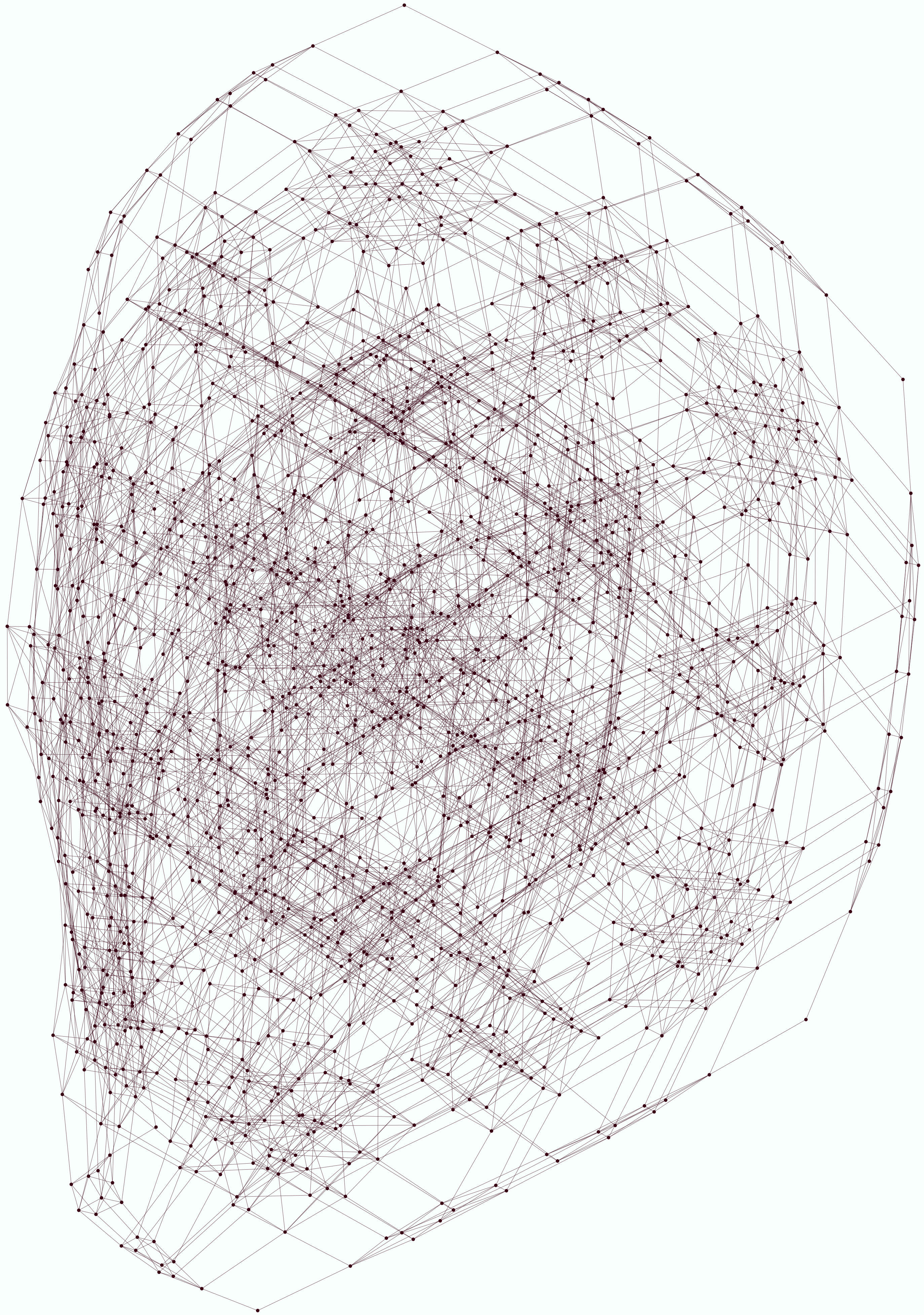


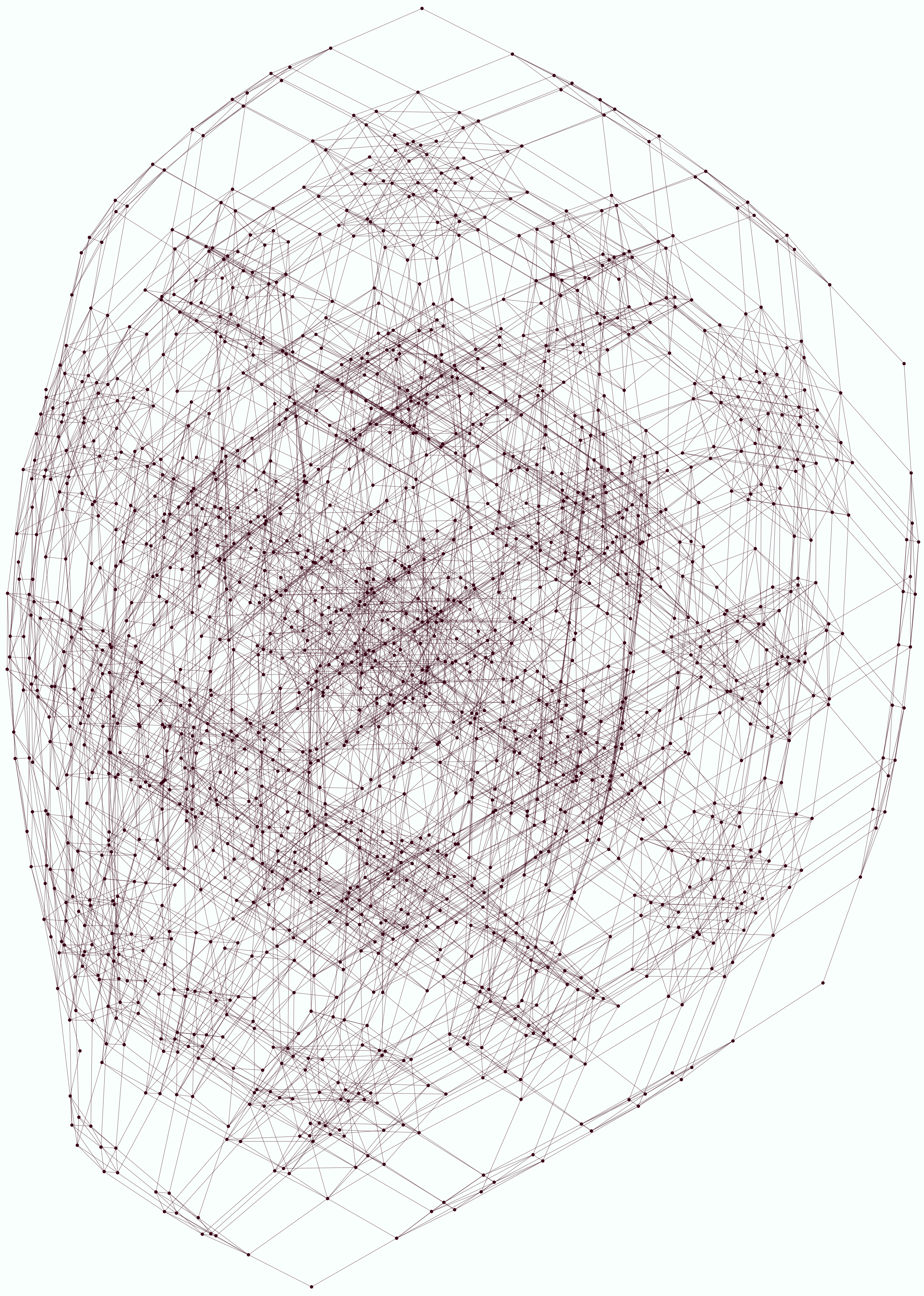


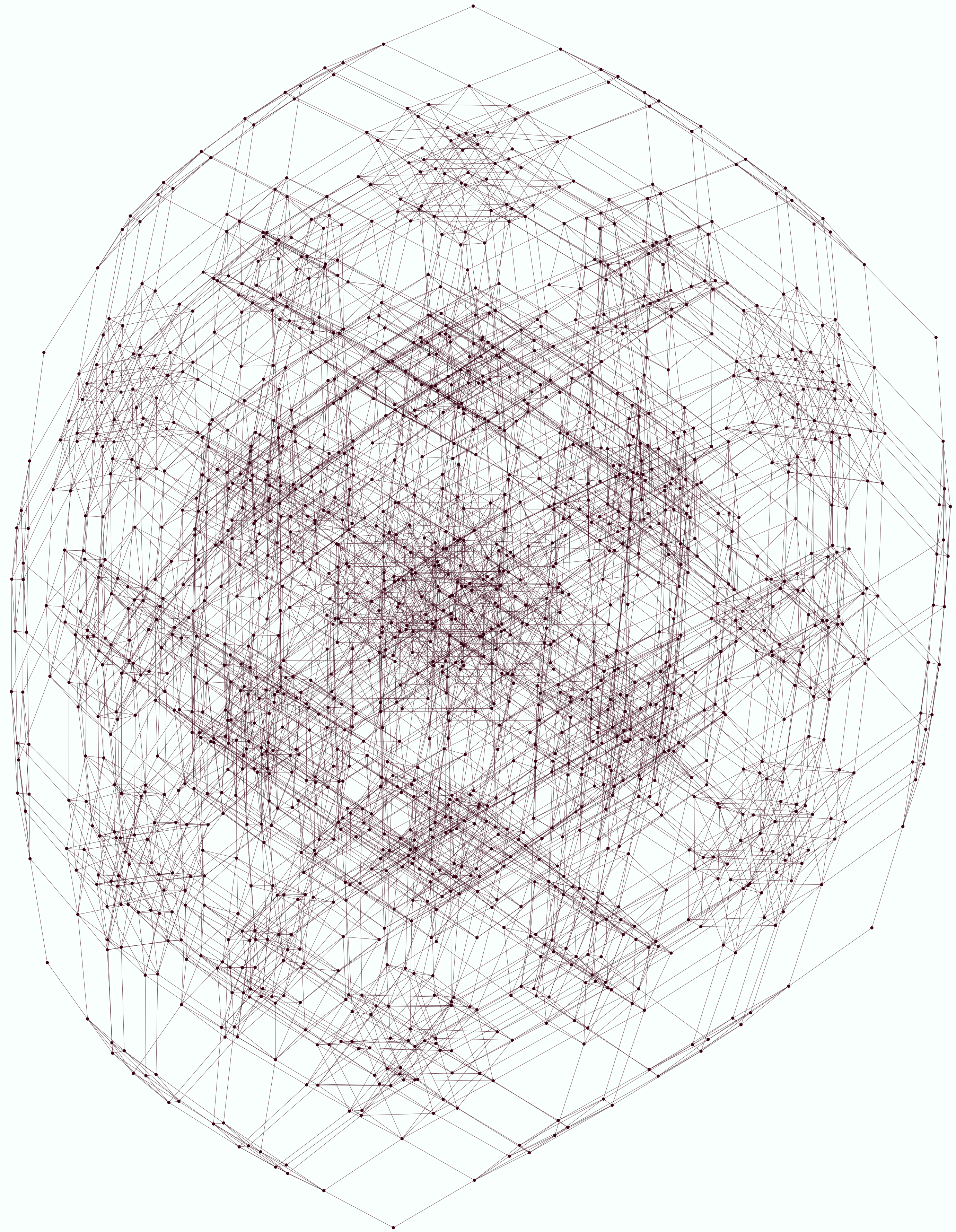


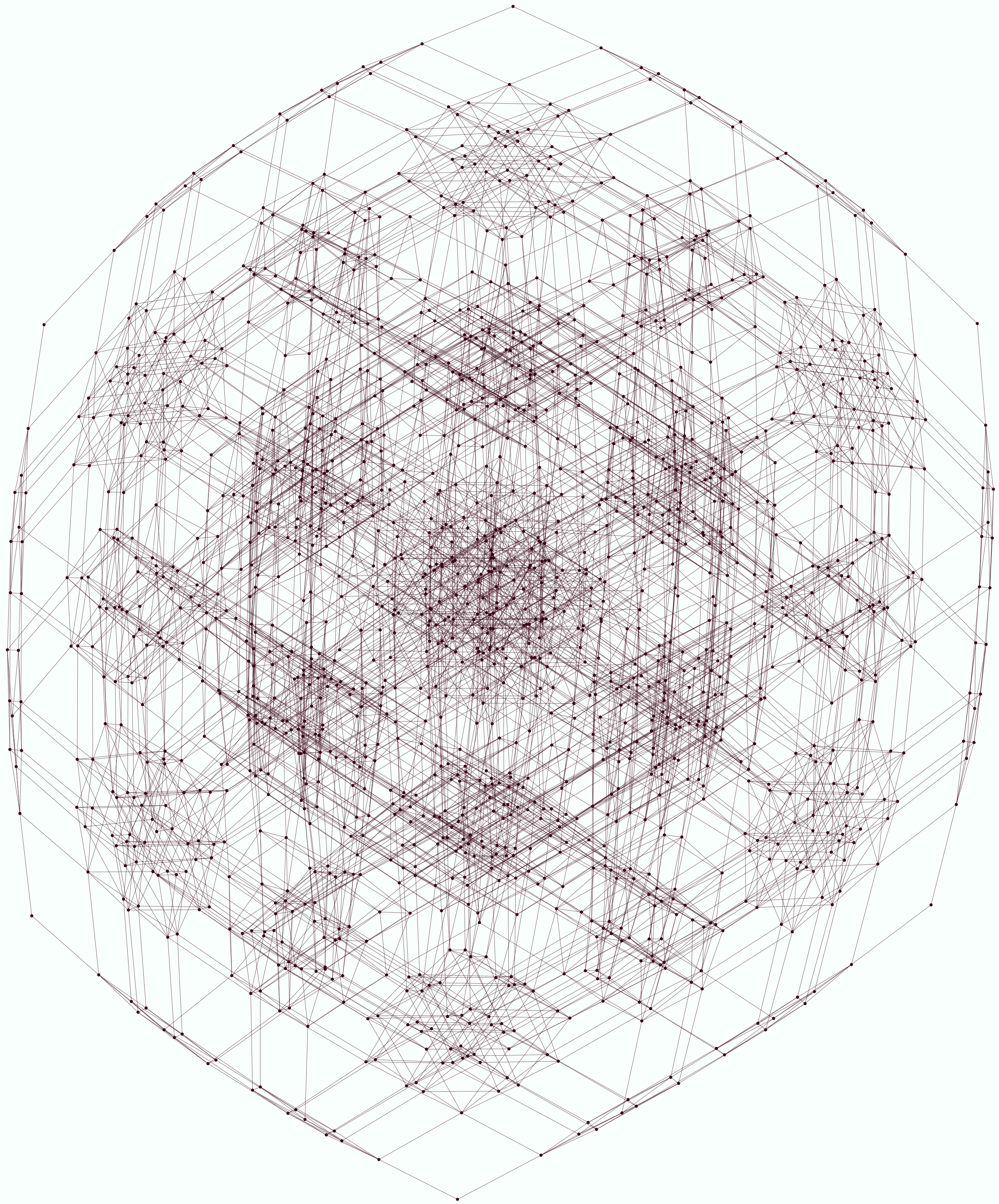


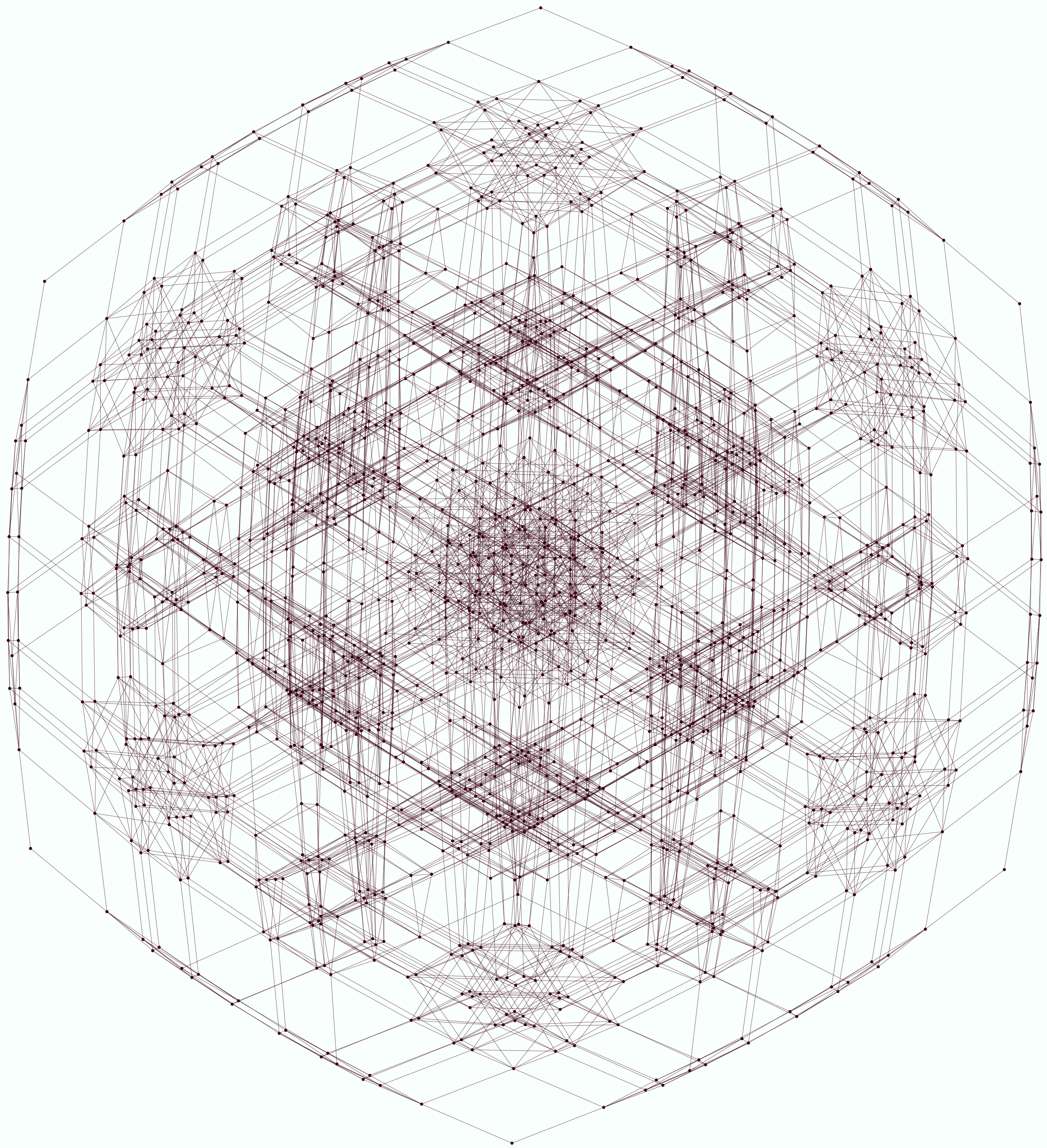


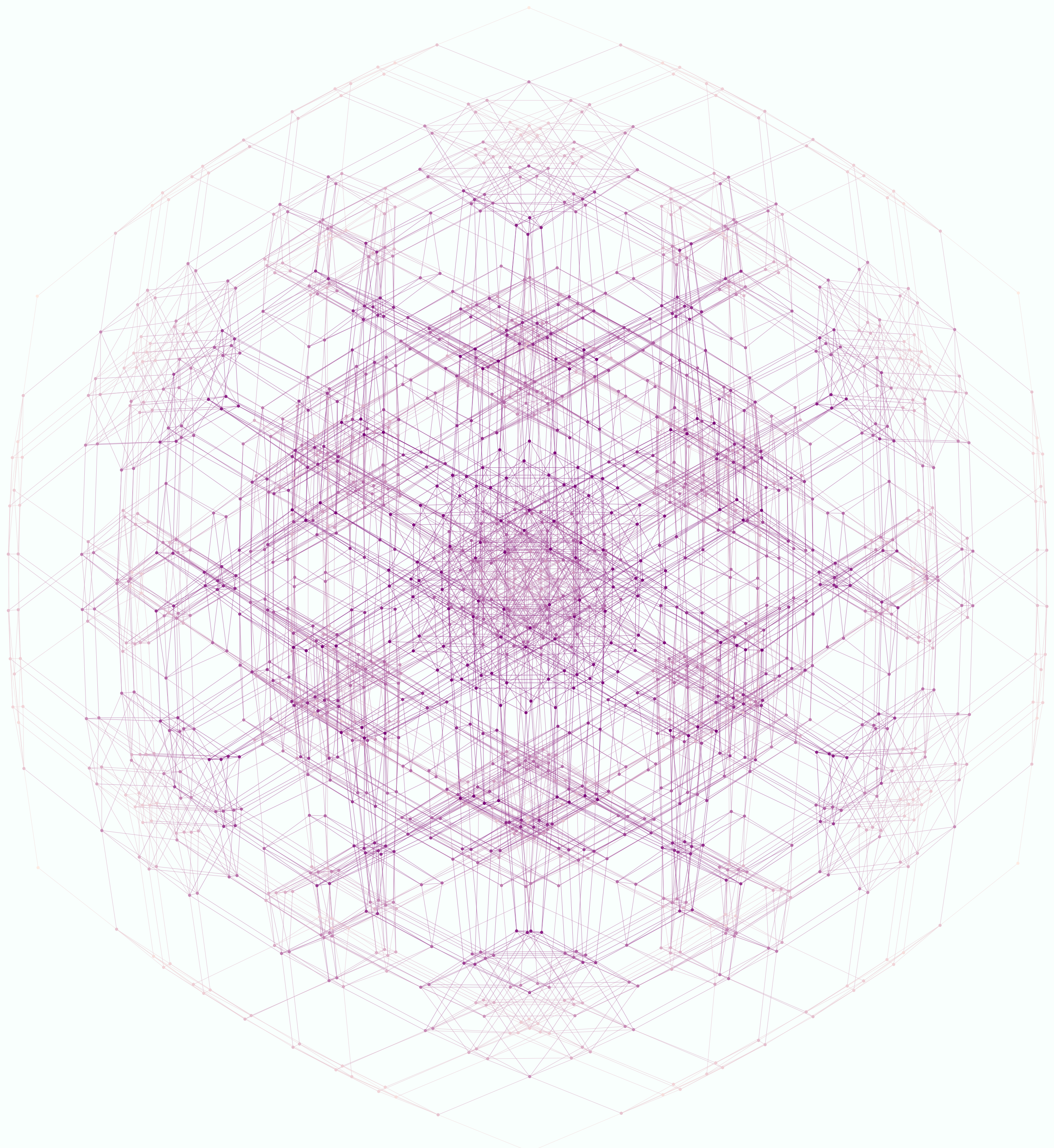


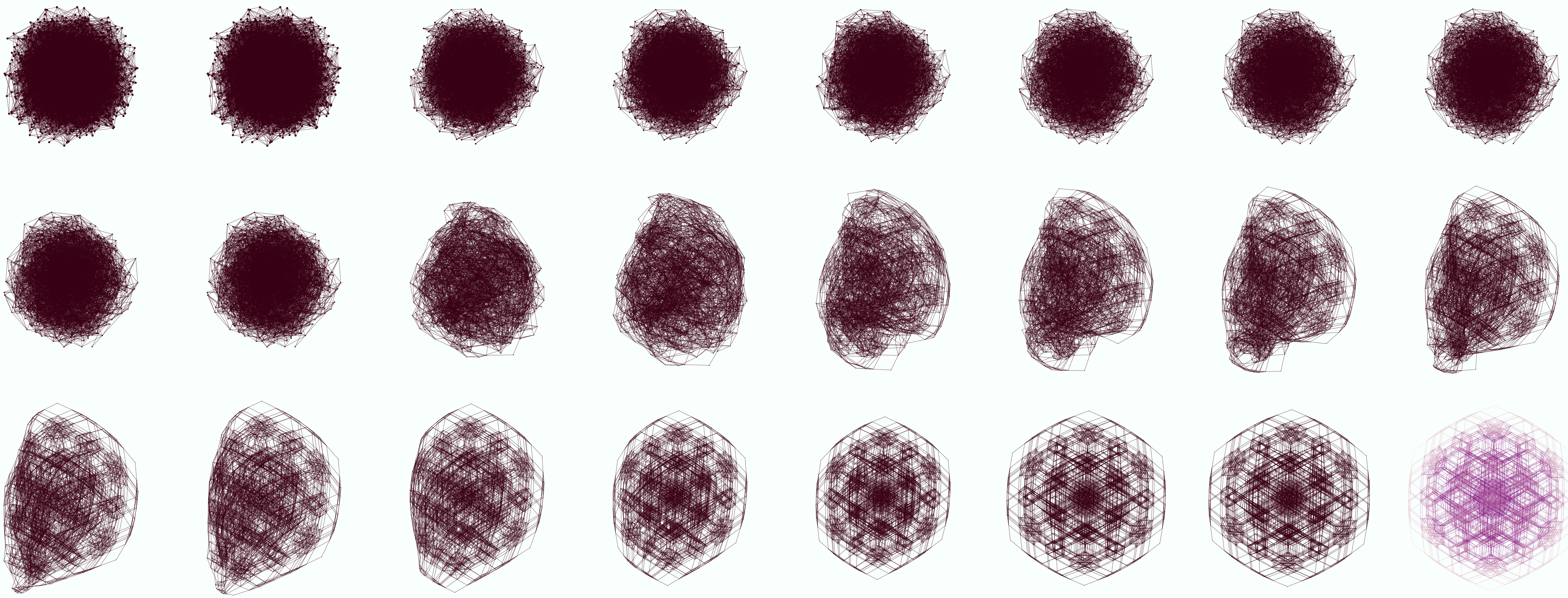


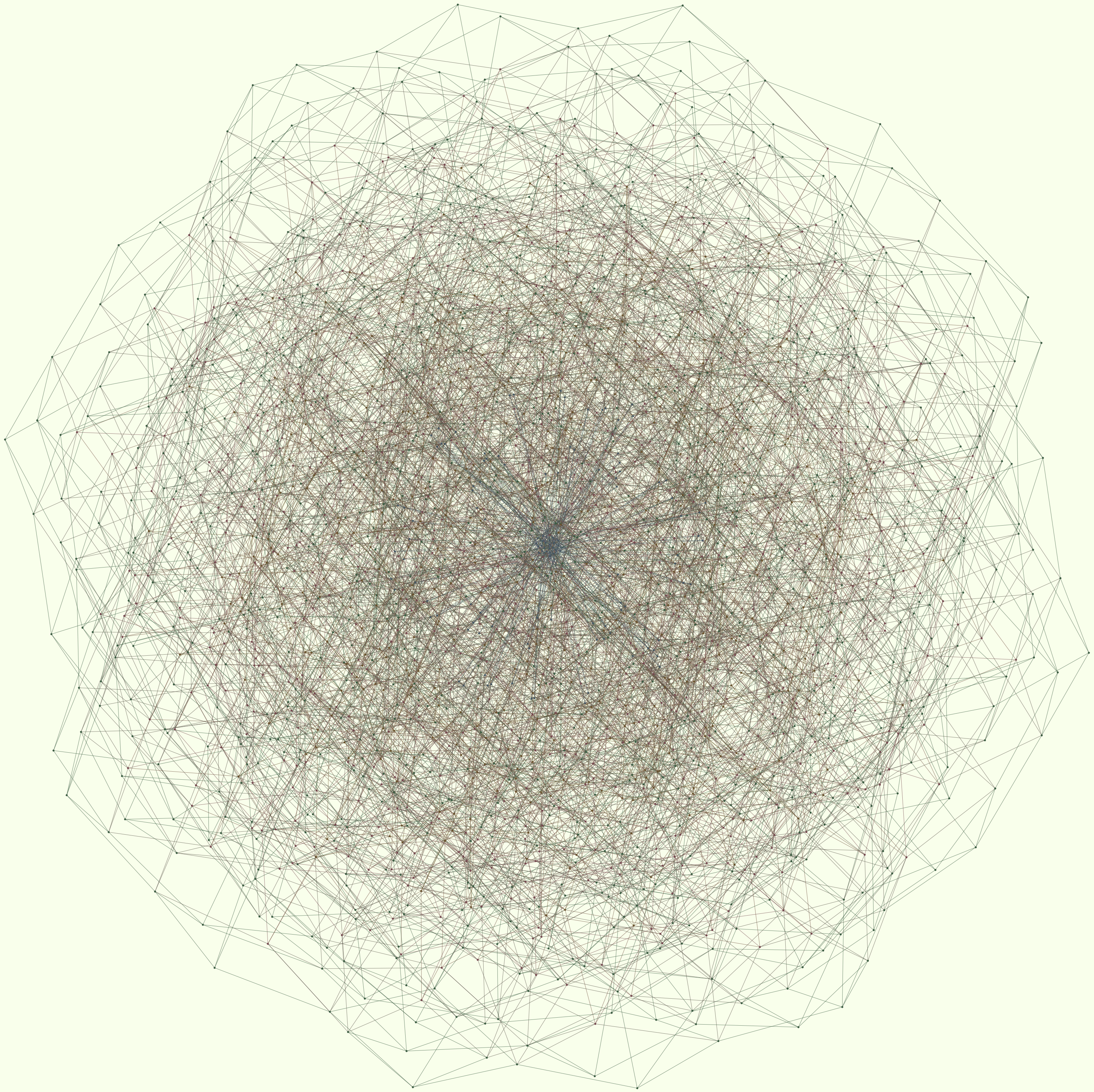


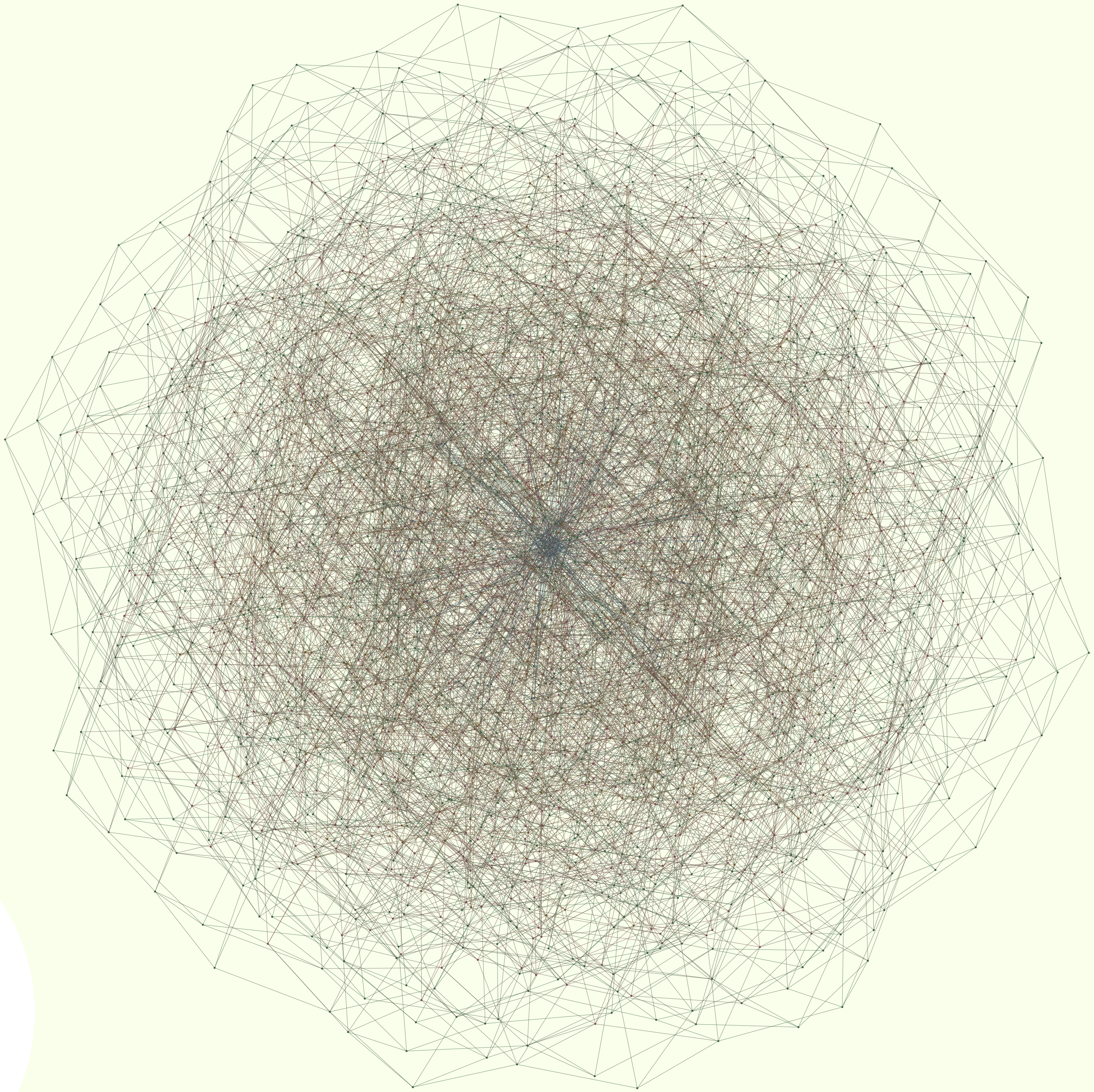












Math talks invariably bring joy.
Sometimes it is when they end.